Solving nonlinear differential equations by analytic iteration

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A method is presented for calculating solutions to differential equations analytically for a variety of problems in physics. An iteration procedure based on the recently proposed BLUES (Beyond Linear Use of Equation Superposition) function method [1] is shown to converge for nonlinear differential equations of diverse types. Case studies are presented for solitary wave solutions of the Camassa-Holm equation and for traveling wavefront solutions of the Burgers equation, with source terms. The convergence of the analytical approximations towards the numerically exact solution is exponentially rapid. In practice, the zeroth-order approximation (a simple convolution) is already useful and the first-order approximation is already accurate while still easy to calculate. The type of nonlinearity can be chosen rather freely, which makes the method generally applicable.

Reference

[1] J.O. Indekeu and K.K. Müller-Nedebock, "BLUES function method in computational physics", J. Phys. A: Math. Theor. **51**, 165201 (2018).