Perturbative and colorful lectures on **Strong Interactions**

lecture 3/4

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$$W_{\mu\nu} = -W_1 g_{\mu\nu} + \frac{W_2}{m_p^2} p_\mu p_\nu + \frac{W_3}{m_p^2}(p_\mu q_\nu + q_\mu p_\nu)$$
$$+ \frac{W_4}{m_p^2}(p_\mu q_\nu - q_\mu p_\nu) + \frac{W_5}{m_p^2} q_\mu q_\nu + \frac{W_6}{m_p^2}\varepsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma$$

$W_i(x, Q^2)$

For a single photon exchange, imposing hadronic current conservation and due to the symmetry of $L^{\mu\nu}$, only 2 terms survive.

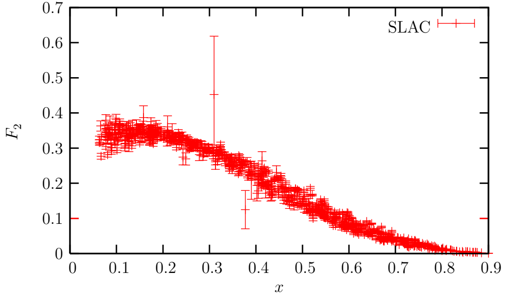
In Björken limit : $Q^2 \to \infty$, $s \to \infty$ and x fixed

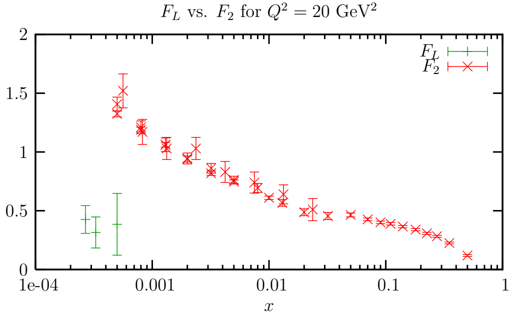
$$\frac{d^2\sigma}{dx\, dQ^2} = \frac{4\,\pi\,\alpha^2}{x\,Q^4}\left[x\,y^2\,F_1(x, Q^2) + (1-y)\,F_2(x, Q^2)\right]$$

with $y = \frac{p \cdot q}{p \cdot k}$ and $x, y \in [0, 1]$, $Q^2 = x\,y\,s$

$$\frac{d^2\sigma}{dx\, dQ^2} = \frac{2\,\pi\,\alpha^2}{x\,Q^4}\left\{\left[1 + (1-y)^2\right] F_2(x, Q^2) - y^2\,F_L(x, Q^2)\right\}$$

SLAC DIS data: scale invariance



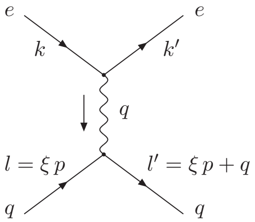


$$F_2(x, Q^2) \to F_2(x)$$

$F_L(x, Q^2)$ small but not zero

DIS in a naive parton model

parton: spin 1/2 point like particle of fractional electric charge e_q



$$\frac{d^2\hat{\sigma}_{eq\to eq}}{dx\,dQ^2}(\xi) = \frac{2\,\pi\,e_q^2\,\alpha^2}{Q^4}\left[1+(1-y)^2\right]\delta(x-\xi)$$

$\Rightarrow$ simple physical interpretation of x : proton momentum fraction carried by the interacting quark

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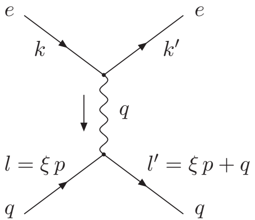

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interacting on the proton is the sum of interaction on all quarks flavours times the probability to find such a quark, integrated over their internal momentum fraction:

$$\frac{d^2\sigma_{ep\to eX}}{dx\,dQ^2} = \int_0^1 d\xi \sum_q f_q(\xi)\frac{d^2\hat{\sigma}_{eq\to eq}}{dx\,dQ^2}(\xi, Q^2)$$

$$= \frac{2\,\pi\,\alpha^2}{Q^4}\int_0^1 d\xi \sum_q e_q^2\, f_q(\xi)\left[1+(1-y)^2\right]\delta(x-\xi)$$

Comparing

$$\frac{d^2\sigma_{ep\to eX}}{dx\,dQ^2} = \frac{2\,\pi\,\alpha^2}{Q^4}\int_0^1 d\xi \sum_q e_q^2\, f_q(\xi)\,\left[y^2 + 2(1-y)\right]\,\delta(x-\xi)$$

to the DIS general expression:

$$\boxed{\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\,\pi\,\alpha^2}{x\,Q^4}\left[x\,y^2\,F_1(x,Q^2) + (1-y)\,F_2(x,Q^2)\right]}$$

one gets a simple interpretation of the SF in the naive quark model:

$$F_1(x,Q^2) = \frac{1}{2}\int_0^1 d\xi \sum_q e_q^2\, f_q(\xi)\,\delta(x-\xi) = \frac{1}{2}\sum_q e_q^2\, f_q(x)$$

$$F_2(x,Q^2) = \int_0^1 d\xi \sum_q e_q^2\, x\, f_q(\xi)\,\delta(x-\xi) = \sum_q e_q^2\, x\, f_q(x)$$

⇒ simple interpretation:

$$F_1(x,Q^2) = \frac{1}{2}\sum_q e_q^2\, f_q(x) = \frac{1}{2}\sum_q e_q^2\, [q(x)+\bar{q}(x)] = F_1(x),$$

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⇒ scale invariance

$\Rightarrow$ simple interpretation:

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$\Rightarrow$ scale invariance

Callan-Gross relation:

$$F_2(x) = 2\,x\,F_1(x) \qquad \Rightarrow \qquad F_L(x) = F_2(x) - 2\,x\,F_1(x) = 0\,.$$

(due to helicity conservation: F_L is associated to spin flip)

very nice success - but not the full story

Using protons and neutrons

Assumption ($SU(2)$ isospin): neutron is just proton with $u \Leftrightarrow d$:

proton = uud; neutron = ddu

Isospin: $u_n(x) = d_p(x)\,, \qquad d_n(x) = u_p(x)$

$$F_2^p = \frac{4}{9}u_p(x) + \frac{1}{9}d_p(x)$$
$$F_2^n = \frac{4}{9}u_n(x) + \frac{1}{9}d_n(x) = \frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)$$

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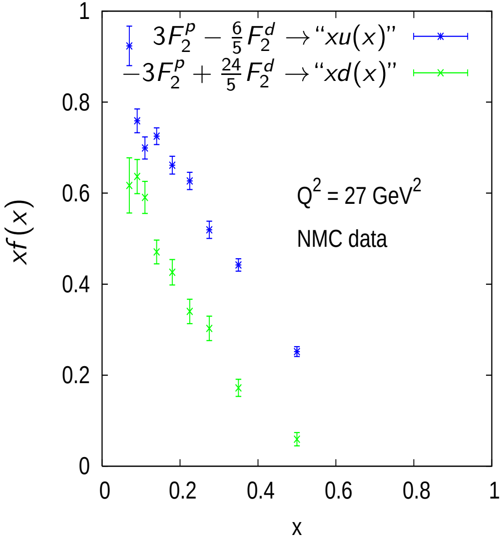
Linear combinations of F_2^p and F_2^n give separately $u_p(x)$ and $d_p(x)$.

Experimentally, get F_2^n from deuterons: $F_2^d = \frac{1}{2}(F_2^p + F_2^n)$

NMC proton & deuteron data



Combine F_2^p & F_2^d data, deduce $u(x)$, $d(x)$:

- Definitely more up than down (✓)

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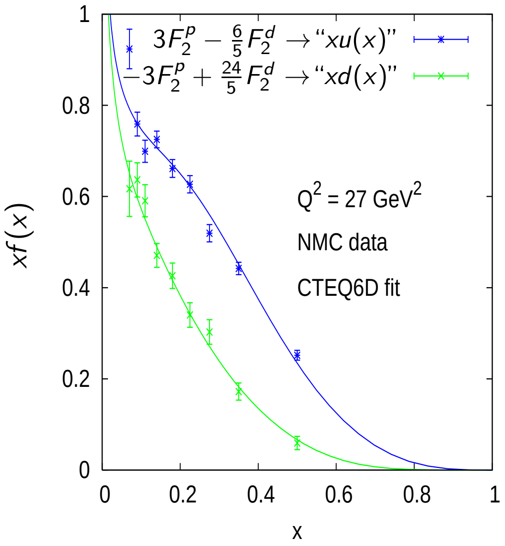

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How much u and d?

- Total $U = \int dx\, u(x)$
- $u(x) \sim d(x) \sim x^{-1.25}$

non-integrable divergence

So why do we say proton $=$ uud?

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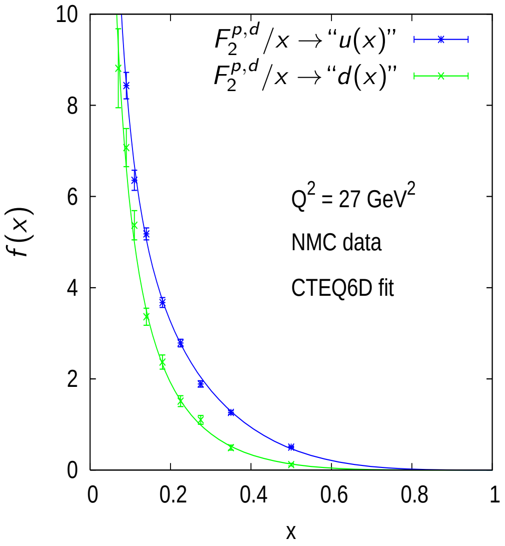

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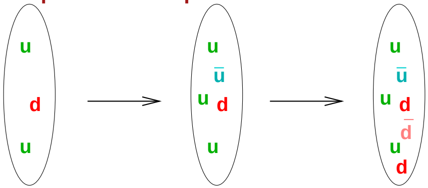
Anti-quarks in proton



How can there be infinite number of quarks in proton?

Proton wave function *fluctuates* — extra $u\bar{u}$, $d\bar{d}$ pairs (*sea quarks*) can appear:

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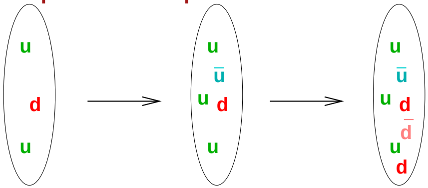

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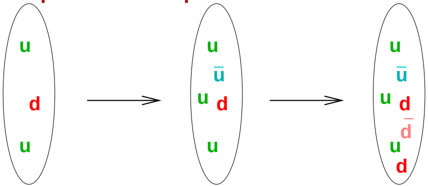
Anti quarks also have distributions, $\bar{u}(x)$, $\bar{d}(x)$

$$F_2 = \frac{4}{9}(xu(x) + x\bar{u}(x)) + \frac{1}{9}(xd(x) + x\bar{d}(x))$$

NB: photon interaction $\sim$ square of charge

Anti-quarks in proton



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- Previous transparency: we were actually looking at $\sim u + \bar{u}$, $d + \bar{d}$
- Number of extra quark-antiquark pairs can be *infinite,* so

$$\int dx\,(u(x) + \bar{u}(x)) = \infty$$

as long as they carry little momentum (mostly at low x)

When we say proton has 2 up quarks & 1 down quark, we mean

$$\int dx\,(u(x) - \bar{u}(x)) = 2\,, \qquad \int dx\,(d(x) - \bar{d}(x)) = 1$$

$u - \bar{u} = u_V$ is known as a *valence* distribution.

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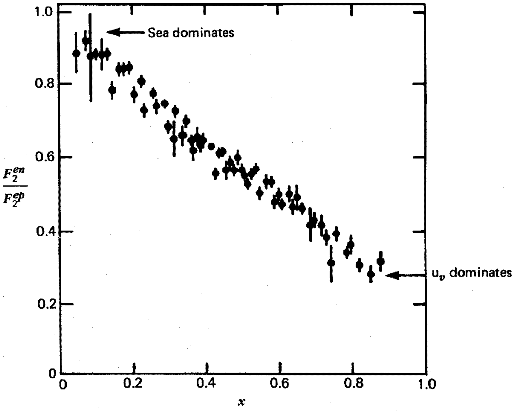
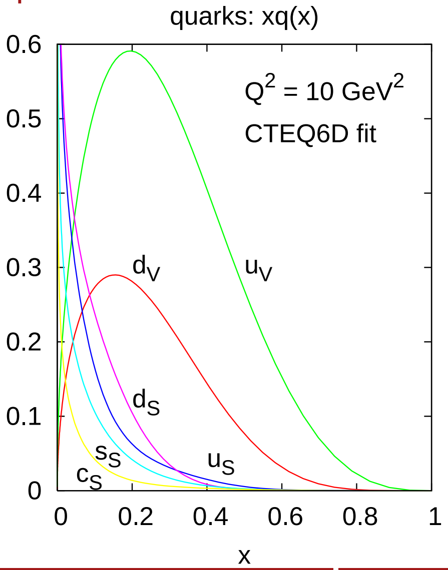


$$\lim_{x\to 1} \quad \frac{F_2^n}{F_2^p} \to \frac{u_V + 4d_V}{4u_V + d_V} \simeq \frac{1}{4}$$

$$\lim_{x\to 0} \quad \frac{F_2^n}{F_2^p} \to 1$$

All quarks



These & other methods → whole set of quarks & anti quarks

NB: also strange and charm quarks

- valence quarks ($u_V = u - \bar{u}$) are *hard*
  $x \to 1 : xq_V(x) \sim (1-x)^3$
  $x \to 0 : xq_V(x) \sim x^{0.5}$
- sea quarks ($u_S = 2\bar{u}, \ldots$) fairly *soft* (low-momentum)
  $x \to 1 : xq_S(x) \sim (1-x)^7$
  $x \to 0 : xq_S(x) \sim x^{-0.2}$

Momentum sum rule

Check momentum sum-rule (sum over all species carries all momentum):

$$\sum_i \int dx\, x\, q_i(x) = 1$$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

Where is missing momentum?

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Where is missing momentum?

Only parton type we've neglected so far is the

gluon

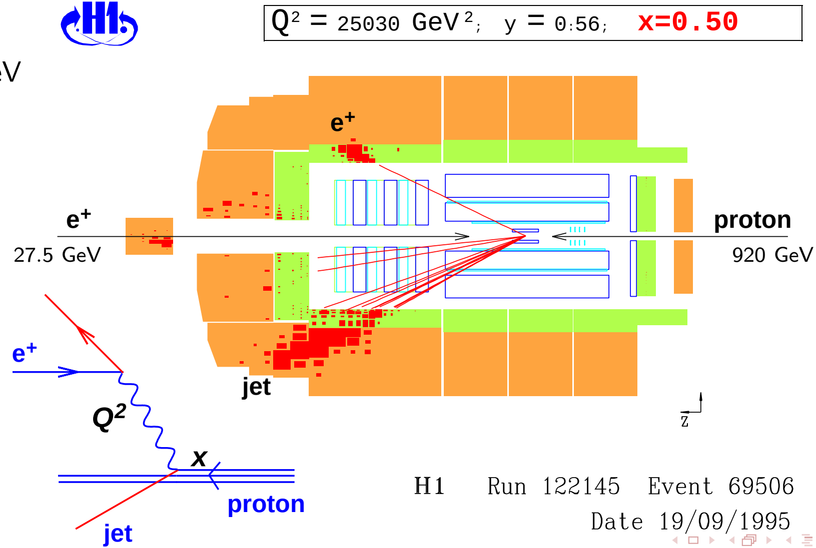
Not directly probed by γ.

To discuss gluons we must go beyond 'naive' leading order picture, and bring in QCD effects. . .

How to measure Structure Functions

HERA

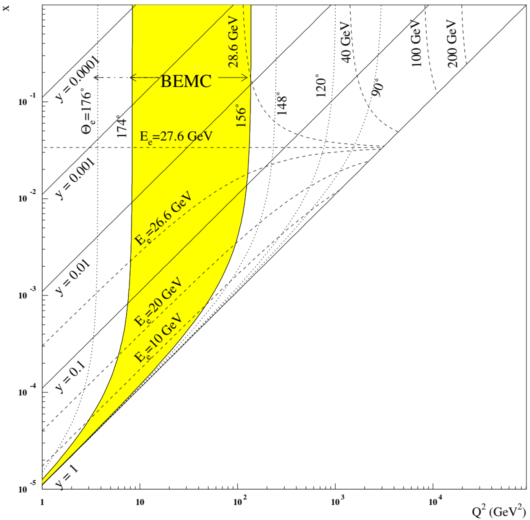
$\sqrt{s} = 318$ GeV



Cross section measurement
- Kinematic reconstruction

$$Q_e^2 = 2\,E_e^0\,E_e\,(1+\cos\theta_e)$$

$$x_e = \frac{E_e^0}{E_p^0}\,\frac{E_e\,(1+\cos\theta_e)}{2\,E_e - E_e^0\,(1-\cos\theta_e)}$$



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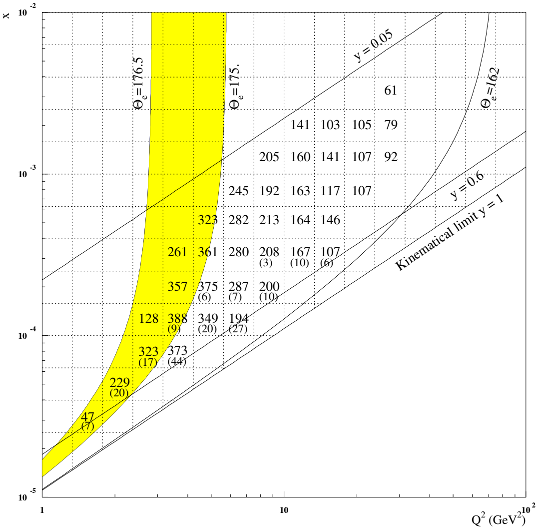
- Make a (x, Q^2) binning

- Count the number of events per bin



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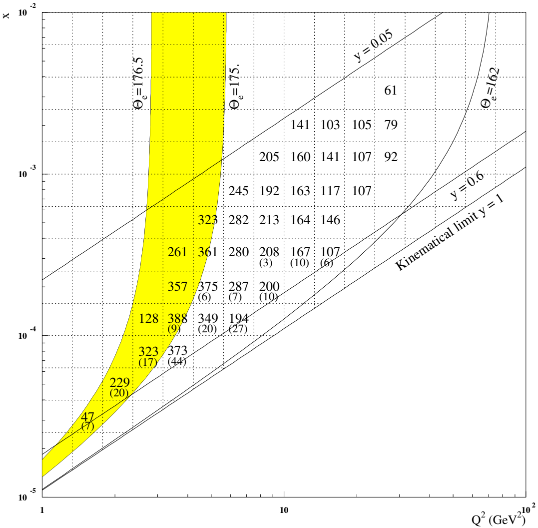
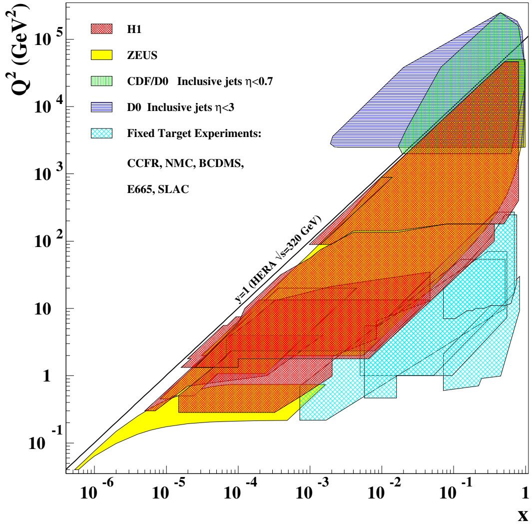
$$x_e = \frac{E_e^0}{E_p^0}\,\frac{E_e\,(1+\cos\theta_e)}{2\,E_e - E_e^0\,(1-\cos\theta_e)}$$

- Make a (x, Q^2) binning
- Count the number of events per bin
  - Compute the cross section

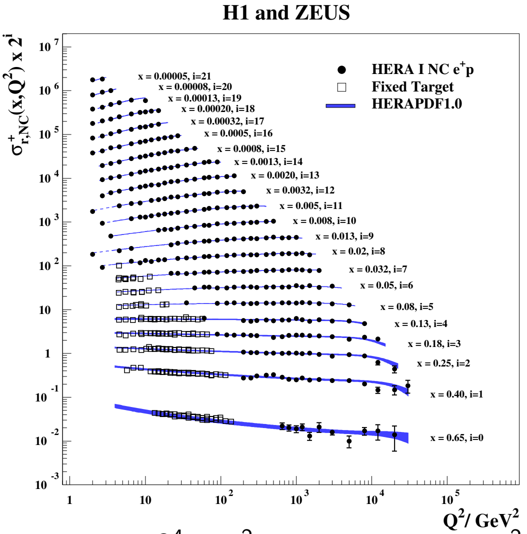
$$\frac{\mathrm{d}\sigma_{Bin}}{\mathrm{d}x\mathrm{d}Q^2} = \frac{N_{bin} - N_{Bg}}{\mathcal{L}\;Acc\;\epsilon}$$



DIS data kinematic domain



DIS measurement



$$\sigma_r(x, Q^2) = \frac{1}{Y_+} \frac{x\, Q^4}{2\,\pi\,\alpha^2} \frac{d^2\sigma}{dx\, dQ^2} = F_2(x, Q^2) - \frac{y^2}{Y_+} F_L(x, Q^2).$$

Effect of Z exchange

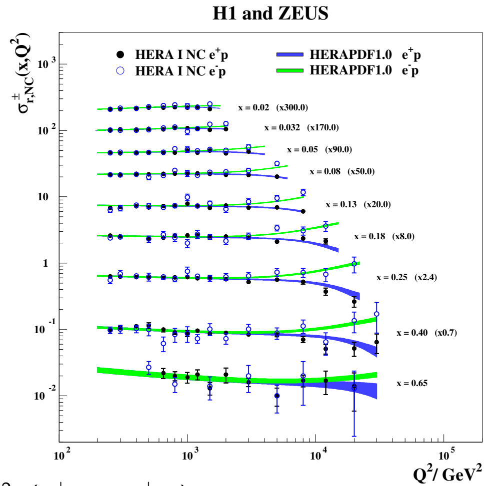
$$\frac{d^2\sigma(e^\pm p \to e^\pm X)}{dx\, dQ^2} = \frac{4\,\pi\,\alpha^2}{x\,Q^4}\Big[x\,y^2\,F_1^{NC}(x,Q^2) + (1-y)\,F_2^{NC}(x,Q^2) \mp y(1-y/2)F_3^{NC}(x,Q^2)\Big].$$

$$F_2^{NC}(x,Q^2) = 2xF_1^{NC}(x,Q^2) \sim \sum_q x\,[q(x)+\bar{q}(x)]$$

$$x\,F_3^{NC}(x,Q^2) \sim \sum_q x[q(x)-\bar{q}(x)]$$

$\Rightarrow$ allows to separate the valence and the sea

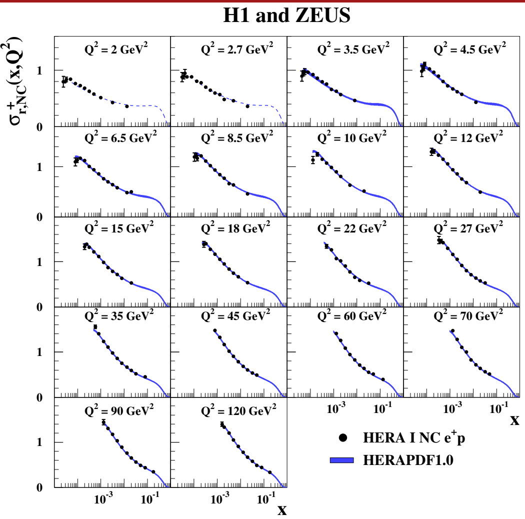
DIS measurement



$$\frac{d^2\sigma(e^\pm p \to e^\pm X)}{dx\, dQ^2} \sim \quad \ldots \mp y(1-y/2)F_3(x,Q^2)$$

$$x\, F_3 \sim \sum_q x[q(x) - \bar{q}(x)]$$

DIS measurement



$Q^2 = s\,x\,y$

$$\sigma_r(x,Q^2) = \frac{1}{Y_+}\,\frac{x\,Q^4}{2\,\pi\,\alpha^2}\,\frac{d^2\sigma}{dx\,dQ^2} = F_2(x,Q^2) - \frac{y^2}{Y_+}\,F_L(x,Q^2)$$

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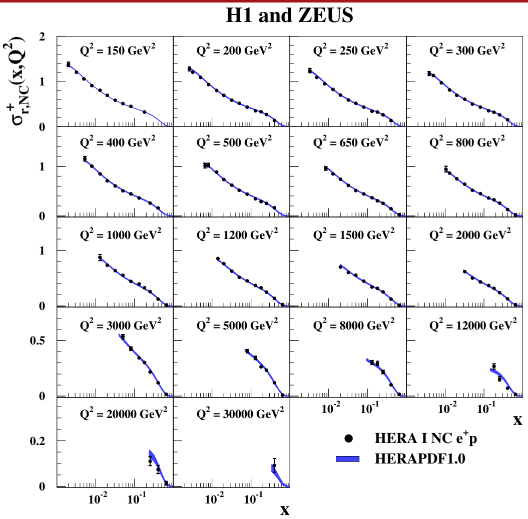

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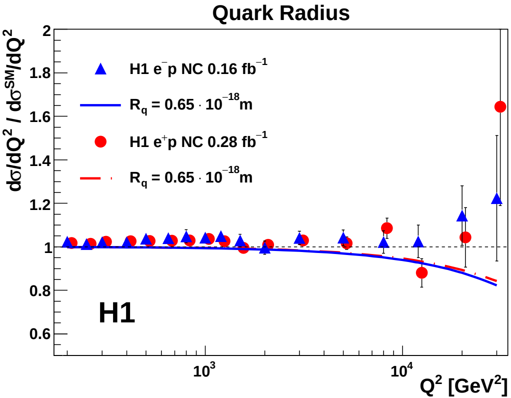
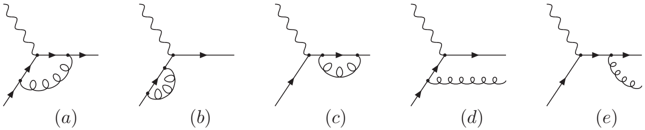
limit on quark size



⇒ limit on quark size: $R_q < 0.65\ 10^{-18}\mathrm{m}$

To be compared to $R_e < 0.28\ 10^{-18}\mathrm{m}$ (and $R_e < 10^{-22}\mathrm{m}$ from a single e in a Penning Trap exp.[Dehmelt 1988]).

First order in α_S



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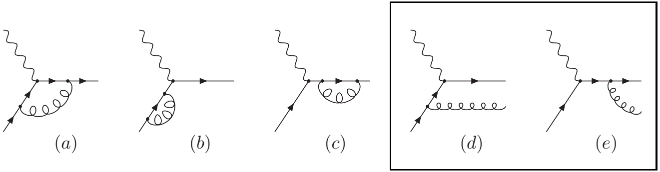

First order in α_S

$$\overline{|\mathcal{M}|^2} = 32\pi^2(e_q^2\alpha\alpha_S)\frac{4}{3}\left[-\frac{t}{s} - \frac{s}{t} + \frac{2u\,Q^2}{st}\right]$$

$$\begin{aligned} t &= (k-p)^2 = (q-k')^2 = -2pk(1-\cos\theta_{qg}) \\ s &= (k+q)^2 = (k'+p)^2 = \ 2pk'(1-\cos\theta_{q'g}) \\ u &= (q-p)^2 = (k-k')^2 \end{aligned}$$

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double pole structure in t (left diag.) and in s (right diag.)

$t \to 0 : E_g \to 0$ or $g \parallel q$

$s \to 0 : E_g \to 0$ or $g \parallel q'$

First order in α_S: gluon radiation

At high energy ($-t \ll s$), using ($p_T = k' sin\theta$) :

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2}(z) = \frac{4\pi^2 \alpha e_q^2}{s} \frac{1}{p_T^2} \frac{\alpha_S}{2\pi} P_{qq}(z)$$

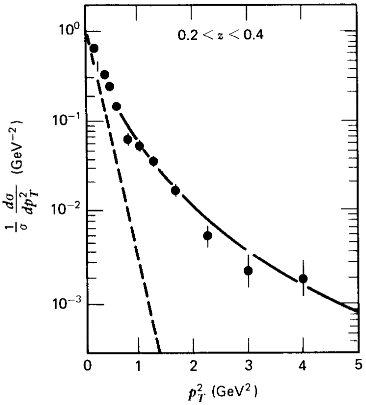
where the longitudinal momentum fraction of the incident quark is:

$$z \equiv \frac{Q^2}{2k \cdot q}$$

$P_{qq}(z)$ is the Splitting Function for a quark that keep a momentum fraction z after a gluon radiation.

$$P_{qq}(z) = \frac{4}{3} \frac{1 + z^2}{1 - z}$$

EMC $\nu N \to \mu X$ measurement [1980]



– – –– no gluon radiation included

—— gluon radiation included

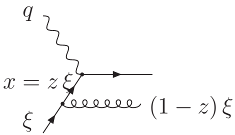
Measurement of the hadron with the largest P_T^2 w.r.t. the virtual photon.

Scaling Violation

These gluon radiation effects have to be included in the quark density.

After integration over p_T^2 between a cutoff (κ) and the maximum ($s/4 = Q^2(1-z)/(4z)$):

$$q(x, Q^2) = q_b(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} q_b(\xi) P_{qq}\left(\frac{x}{\xi}\right) \ln\left(\frac{Q^2}{\kappa^2}\right)$$



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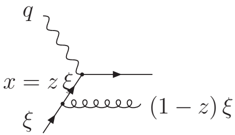

$$
\begin{aligned}
q(x, Q^2) &= q_b(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi}\, q_b(\xi)\, P_{qq}\left(\frac{x}{\xi}\right) \ln\left(\frac{Q^2}{\kappa^2}\right) \\
&= q(x, \mu_F^2) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi}\, q_b(\xi)\, P_{qq}\left(\frac{x}{\xi}\right) \ln\left(\frac{Q^2}{\mu_F^2}\right) \\
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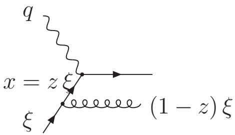
\end{aligned}
$$

$\Rightarrow$ Introduction of the Factorisation scale, μ_F !

Scaling Violation

These gluon radiation effects have to be included in the quark density.

After integration over p_T^2 between a cutoff (κ) and the maximum ($s/4 = Q^2(1-z)/(4z)$):



$$\begin{aligned} q(x,Q^2) &= q_b(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\, q_b(\xi)\, P_{qq}\left(\frac{x}{\xi}\right)\ \ln\left(\frac{Q^2}{\kappa^2}\right) \\ &= q(x,\mu_F^2) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\, q_b(\xi)\, P_{qq}\left(\frac{x}{\xi}\right)\ \ln\left(\frac{Q^2}{\mu_F^2}\right) \\ &= q(x,\mu_F^2) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\, q(\xi,\mu_F^2)\, P_{qq}\left(\frac{x}{\xi}\right)\ \ln\left(\frac{Q^2}{\mu_F^2}\right) + \mathcal{O}(\alpha_s^2) \end{aligned}$$

$\Rightarrow$ Introduction of the Factorisation scale, μ_F !

$\Rightarrow$ The scale invariance is logarithmically broken !

DGLAP equation

$q(x, Q^2)$ being an observable, it cannot depend on μ_F :

$$\frac{\mathrm{d}}{\mathrm{d}\ln \mu_F^2}\, q(x, Q^2) = 0 \quad \Rightarrow \quad \frac{\mathrm{d}\, q(x, Q^2)}{\mathrm{d}\ln Q^2} = \frac{\alpha_s}{2\,\pi} \int_x^1 \frac{d\xi}{\xi}\, q(\xi, Q^2) P_{qq}\left(\frac{x}{\xi}\right)$$

known as the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equation

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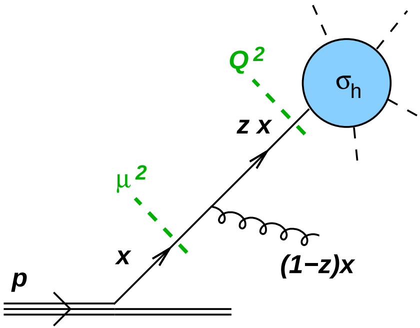

All processes

$$P_{qq}(z) = \frac{4}{3}\frac{1+z^2}{1-z}$$

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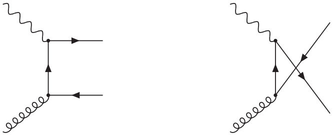
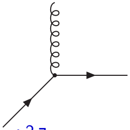
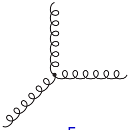


$$P_{qg}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$$



$$P_{gq}(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right]$$



$$P_{gg}(z) \;=\; 6\left[\frac{z}{(1-z)} + (1-z)(z+\frac{1}{z})\right]$$

- P_{qg}, P_{gg}: *symmetric* $z \leftrightarrow 1-z$
- P_{qq}, P_{gg}: *diverge for* $z \to 1$ *(soft gluon emission)*
- P_{gg}, P_{gq}: *diverge for* $z \to 0$

Implies PDFs grow for $x \to 0$

DGLAP equations: flavour structure

Proton contains both quarks and gluons — so DGLAP is a *matrix in flavour space:*

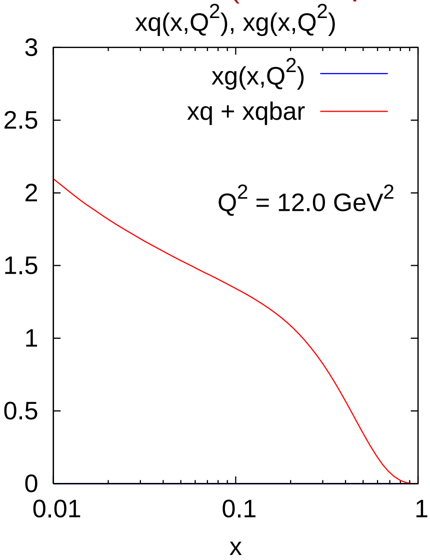
$$\frac{d}{d\ln Q^2}\begin{pmatrix} q_i(x,Q^2) \\ \bar{q}_i(x,Q^2) \\ g(x,Q^2) \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi}\sum_{j=1}^{n_f}\int_x^1 \frac{d\xi}{\xi}\begin{pmatrix} P_{qq}\delta_{ij} & 0 & P_{qg} \\ 0 & P_{gq}\delta_{ij} & P_{qg} \\ P_{gq} & P_{gq} & P_{gg} \end{pmatrix}\begin{pmatrix} q_j(x,Q^2) \\ \bar{q}_j(x,Q^2) \\ g(x,Q^2) \end{pmatrix}$$

$[P_{\bar{q}g} = P_{qg}]$

$$q(x,Q^2) = q(x,\mu_F^2) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\, q(\xi,\mu_F^2)\, P_{qq}\left(\frac{x}{\xi}\right)\ln\left(\frac{Q^2}{\mu_F^2}\right) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\, g(\xi,\mu_F^2)\, P_{qg}\left(\frac{x}{\xi}\right)\ln\left(\frac{Q^2}{\mu_F^2}\right)$$

$\Rightarrow$ measuring the $d\sigma$ one can also access the gluon density !

Effect of DGLAP (initial quarks)



Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{qq} \otimes q$$
$$\partial_{\ln Q^2} g = P_{gq} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

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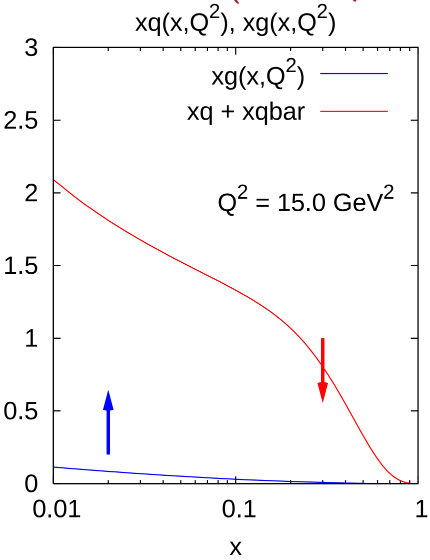

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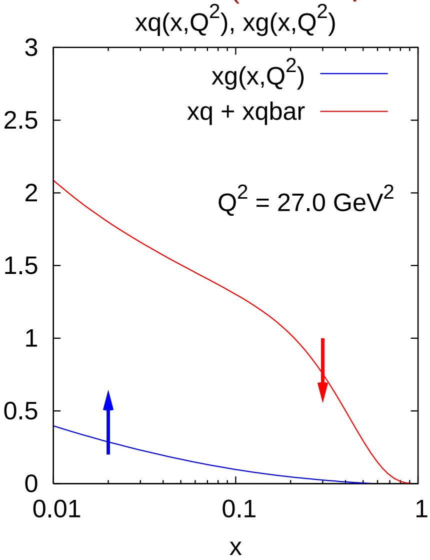

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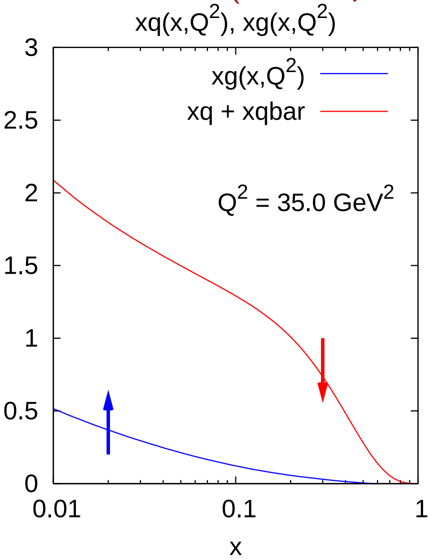

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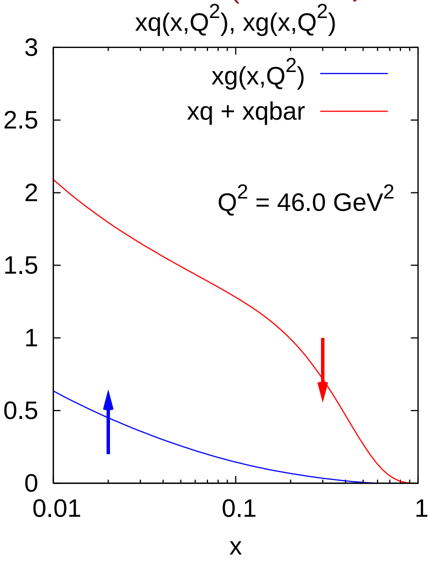

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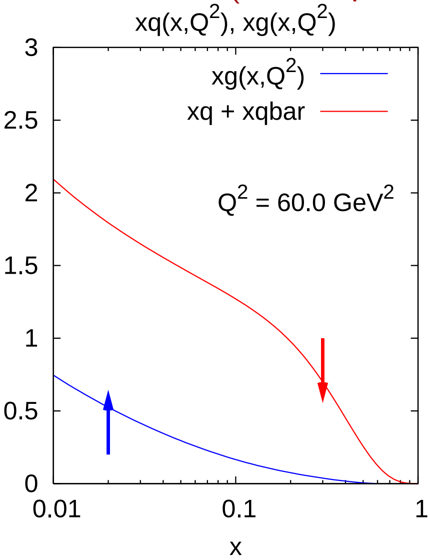

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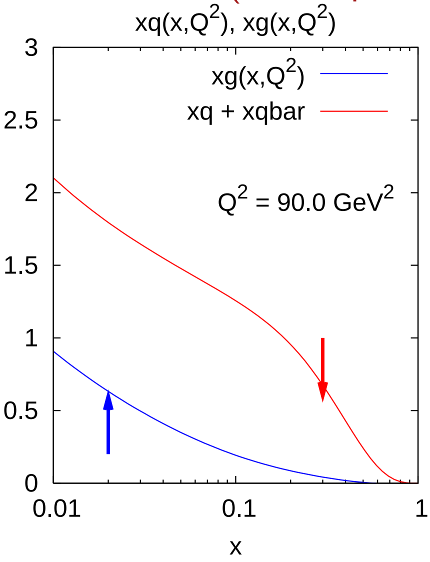

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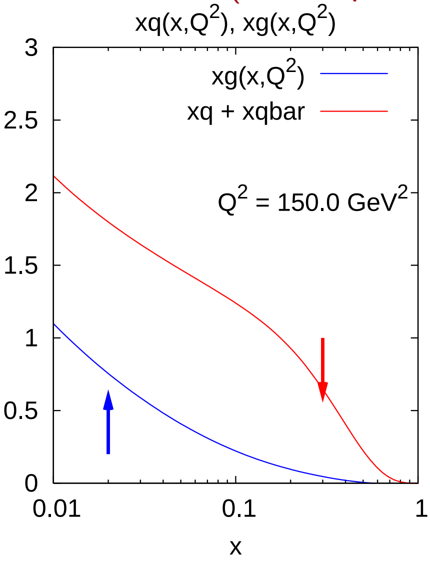

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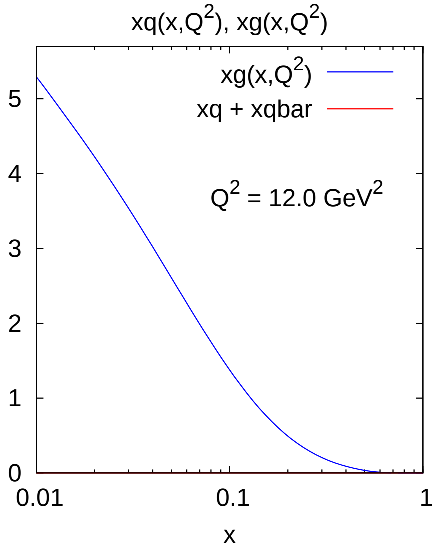

2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{qg} \otimes g$$
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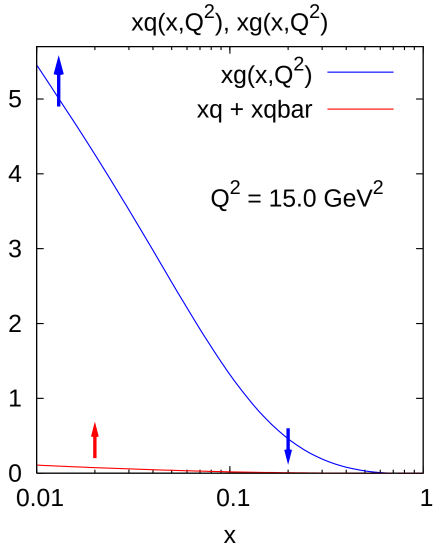

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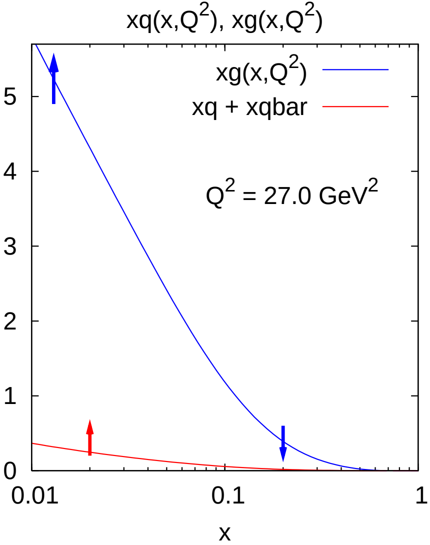

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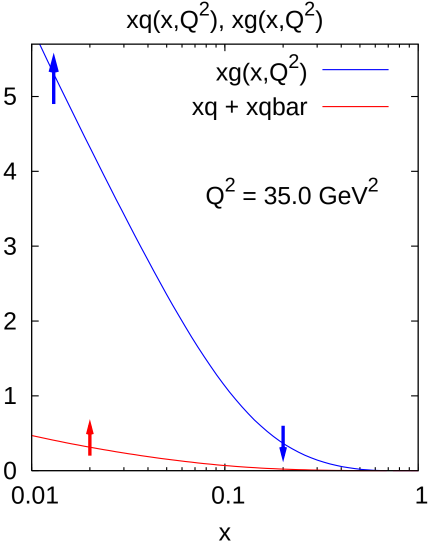

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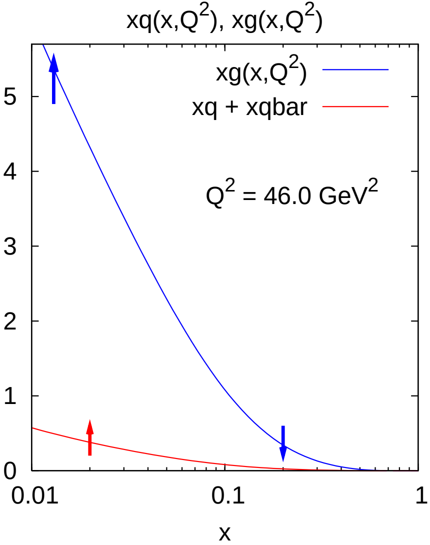

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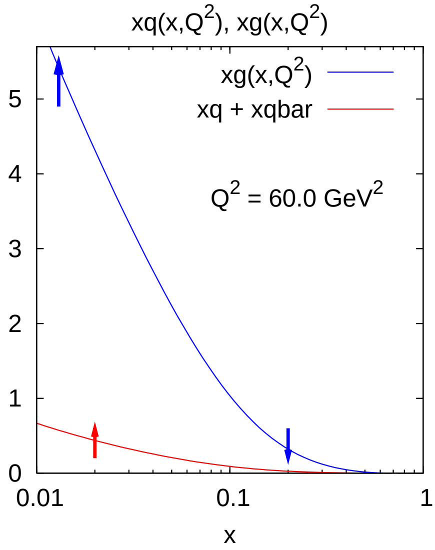

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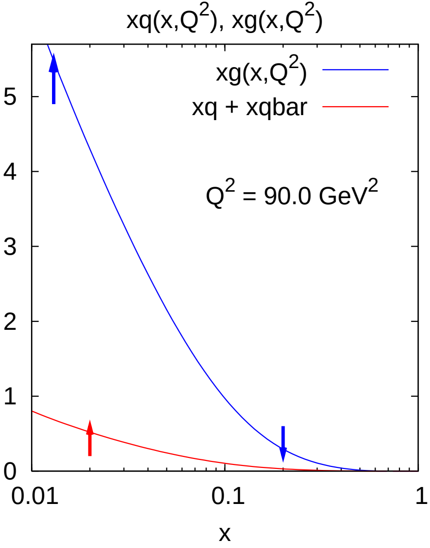

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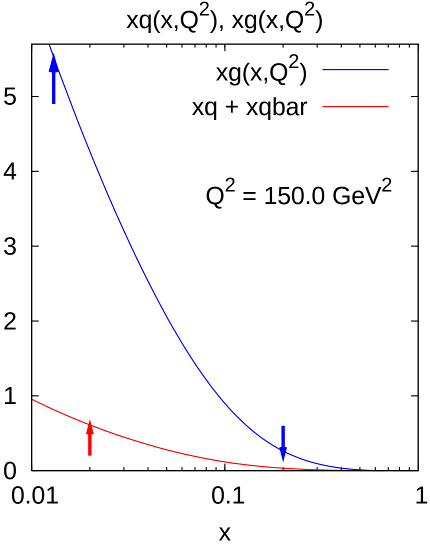

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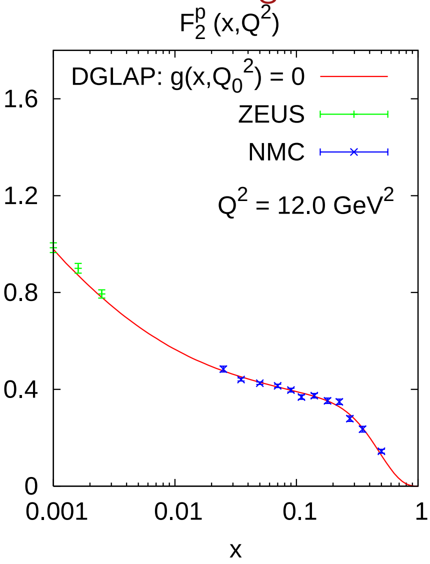
DGLAP evolution

- As Q^2 increases, partons lose longitudinal momentum; distributions all shift to lower x.
- stable point at $x = 0.18$ corresponding to the scaling observed at SLAC
- gluons can be seen because they drive the quark evolution.

Now consider data

DGLAP with initial gluon = 0



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12\mathrm{GeV}^2$.

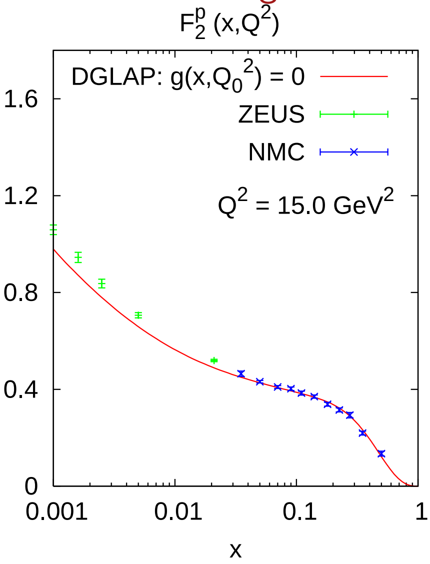
NB: Q_0 often chosen lower

Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

DGLAP with initial gluon $= 0$



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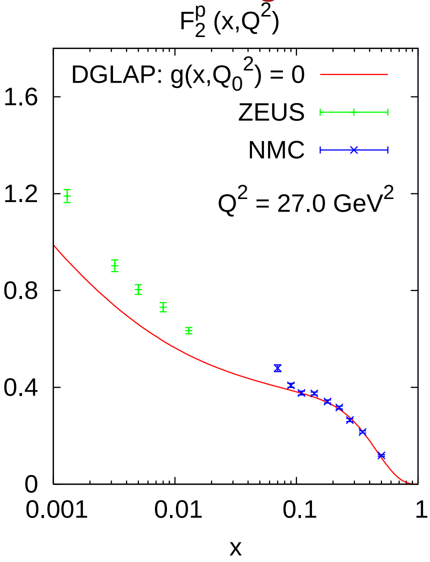

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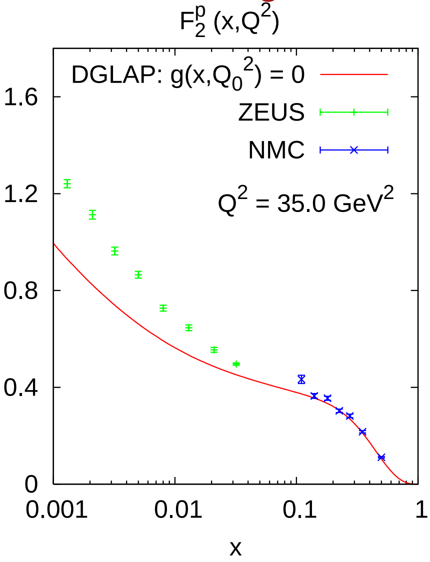

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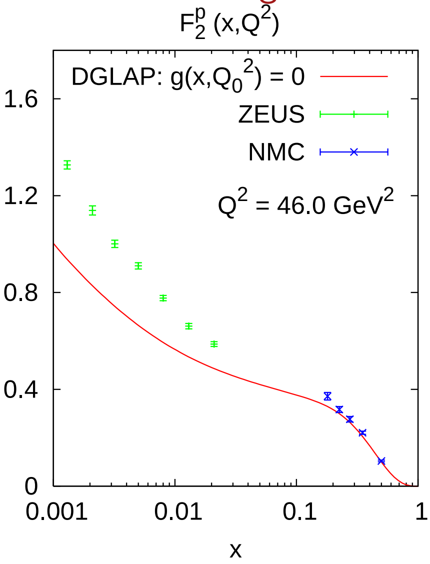

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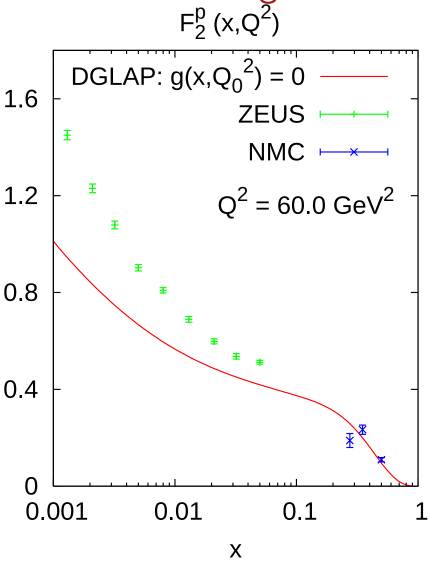

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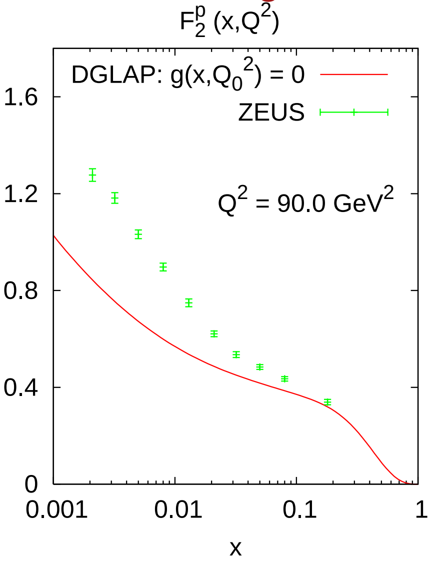

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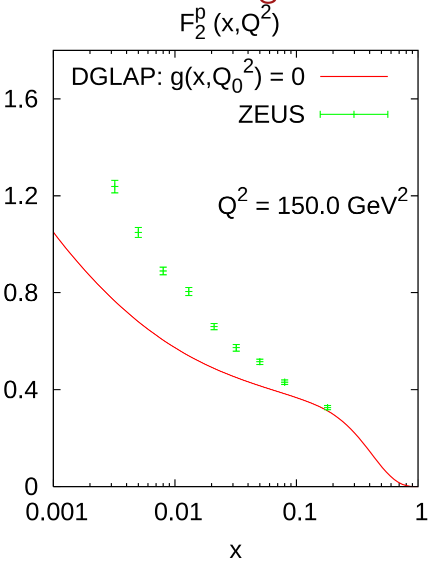

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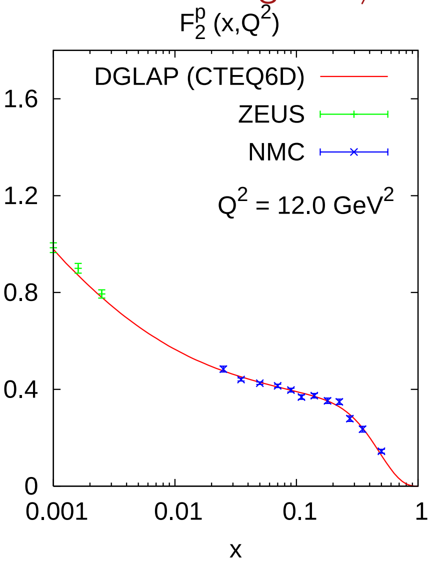
Complete failure!

DGLAP with initial gluon $\neq 0$



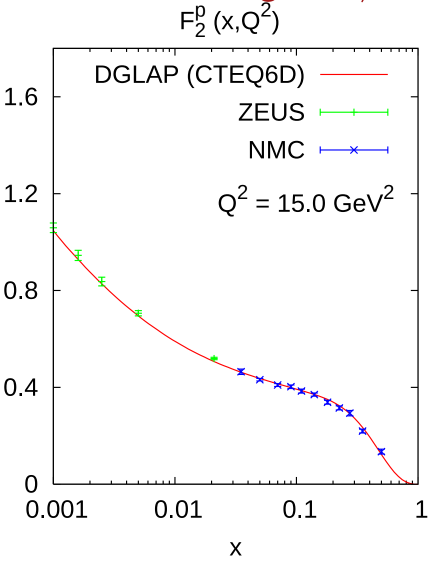
If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, ...
PDF fitting collaborations.

DGLAP with initial gluon $\neq 0$



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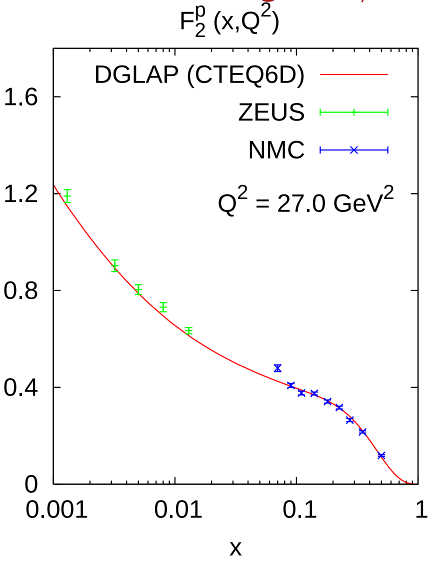

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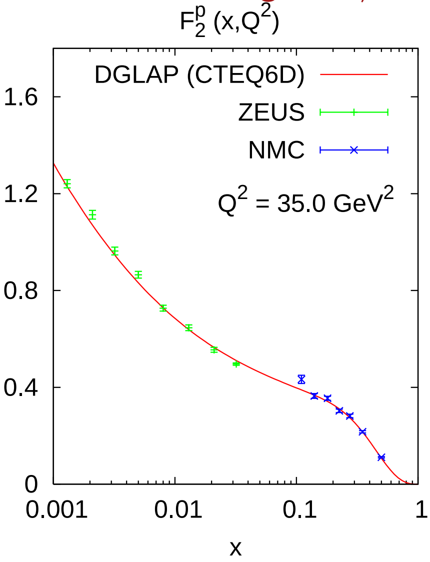

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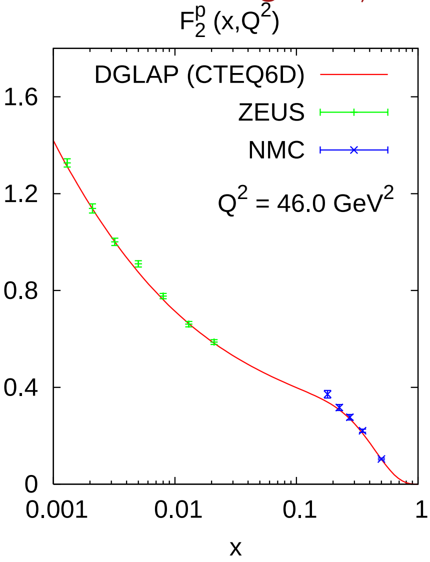

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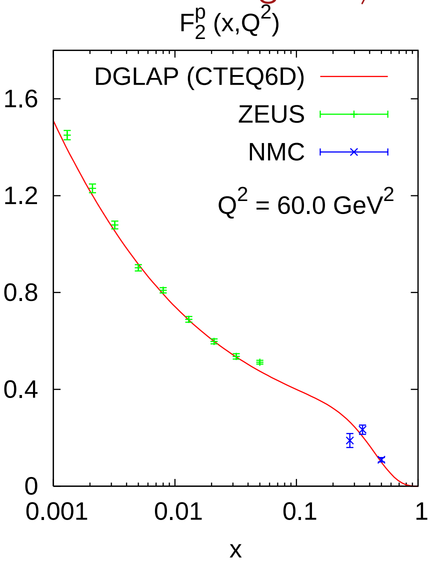

If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, . . .
PDF fitting collaborations.

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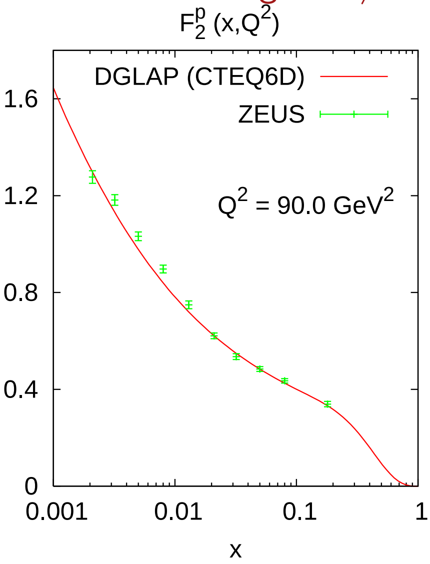

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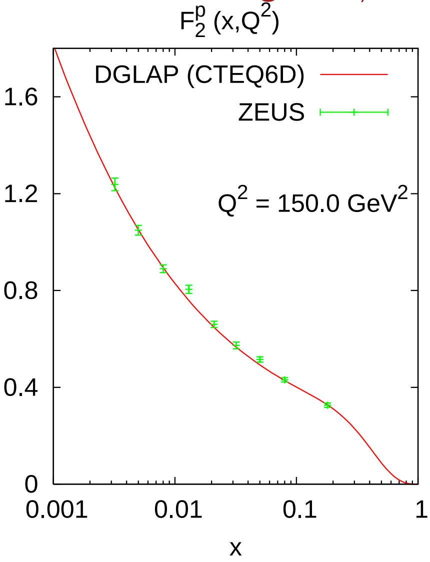

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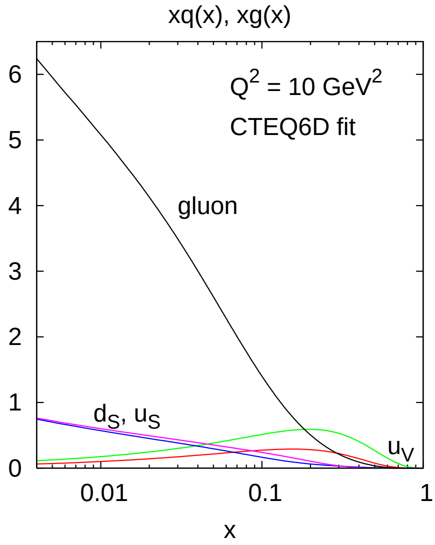
Success!

Gluon distribution



Gluon distribution is **HUGE!**

Can we really trust it?

- Consistency: momentum sum-rule is now *satisfied*.

  NB: gluon mostly at small x

- Agrees with vast range of data

- such a set of q and g densities is called a PDF set for Parton Distribution Functions.

Higher-order calculations

$P_{ab}^{(1)}$: Curci, Furmanski & Petronzio '80

$$P^{(1)}_{\rm ps}(x) = 4\,C_F n_f \left(\frac{20}{9}\frac{1}{x} - 2 + 6x - 4{\rm H}_0 + x^2\left[\frac{8}{3}{\rm H}_0 - \frac{56}{9}\right] + (1+x)\left[5{\rm H}_0 - 2{\rm H}_{0,0}\right]\right)$$

$$P^{(1)}_{\rm qg}(x) = 4\,C_A n_f \left(\frac{20}{9}\frac{1}{x} - 2 + 25x - 2p_{\rm qg}(-x){\rm H}_{-1,0} - 2p_{\rm qg}(x){\rm H}_{1,1} + x^2\left[\frac{44}{3}{\rm H}_0 - \frac{218}{9}\right] + 4(1-x)\left[{\rm H}_{0,0} - 2{\rm H}_0 + x{\rm H}_1\right] - 4\zeta_2 x - 6{\rm H}_{0,0} + 9{\rm H}_0\right) + 4\,C_F n_f \left(2p_{\rm qg}(x)\left[{\rm H}_{1,0} + {\rm H}_{1,1} + {\rm H}_2 - \zeta_2\right] + 4x^2\left[{\rm H}_0 + {\rm H}_{0,0} + \frac{5}{2}\right] + 2(1-x)\left[{\rm H}_0 + {\rm H}_{0,0} - 2x{\rm H}_1 + \frac{29}{4}\right] - \frac{15}{2} - {\rm H}_{0,0} - \frac{1}{2}{\rm H}_0\right)$$

$$P^{(1)}_{\rm gq}(x) = 4\,C_A C_F \left(\frac{1}{x} + 2p_{\rm gq}(x)\left[{\rm H}_{1,0} + {\rm H}_{1,1} + {\rm H}_2 - \frac{11}{6}{\rm H}_1\right] - x^2\left[\frac{8}{3}{\rm H}_0 - \frac{44}{9}\right] + 4\zeta_2 - 2 - 7{\rm H}_0 + 2{\rm H}_{0,0} - 2{\rm H}_1 x + (1+x)\left[2{\rm H}_{0,0} - 5{\rm H}_0 + \frac{37}{9}\right] - 2p_{\rm gq}(-x){\rm H}_{-1,0}\right) - 4\,C_F n_f \left(\frac{2}{3}x - p_{\rm gq}(x)\left[\frac{2}{3}{\rm H}_1 - \frac{10}{9}\right]\right) + 4\,C_F^2 \left(p_{\rm gq}(x)\left[3{\rm H}_1 - 2{\rm H}_{1,1}\right] + (1+x)\left[{\rm H}_{0,0} - \frac{7}{2} + \frac{7}{2}{\rm H}_0\right] - 3{\rm H}_{0,0} + 1 - \frac{3}{2}{\rm H}_0 + 2{\rm H}_1 x\right)$$

$$P^{(1)}_{\rm gg}(x) = 4\,C_A n_f \left(1 - x - \frac{10}{9}p_{\rm gg}(x) - \frac{13}{9}\left(\frac{1}{x} - x^2\right) - \frac{2}{3}(1+x){\rm H}_0 - \frac{2}{3}\delta(1-x)\right) + 4\,C_A^2 \left(27 + (1+x)\left[\frac{11}{3}{\rm H}_0 + 8{\rm H}_{0,0} - \frac{27}{2}\right] + 2p_{\rm gg}(-x)\left[{\rm H}_{0,0} - 2{\rm H}_{-1,0} - \zeta_2\right] - \frac{67}{9}\left(\frac{1}{x} - x^2\right) - 12{\rm H}_0 - \frac{44}{3}x^2{\rm H}_0 + 2p_{\rm gg}(x)\left[\frac{67}{18} - \zeta_2 + {\rm H}_{0,0} + 2{\rm H}_{1,0} + 2{\rm H}_2\right] + \delta(1-x)\left[\frac{8}{3} + 3\zeta_3\right]\right) + 4\,C_F n_f \left(2{\rm H}_0 + \frac{2}{3}\frac{1}{x} + \frac{10}{3}x^2 - 12 + (1+x)\left[4 - 5{\rm H}_0 - 2{\rm H}_{0,0}\right] - \frac{1}{2}\delta(1-x)\right) .$$

NLO:

$$P_{ab} = \frac{\alpha_s}{2\pi}P_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ab}^{(1)}$$

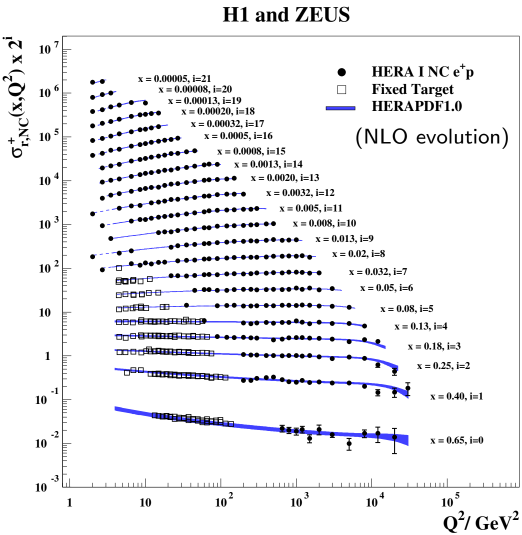
NNLO splitting functions

$P_{ab}^{(2)}$: Moch, Vermaseren & Vogt '04

NNLO:

$$P_{ab} = \frac{\alpha_s}{2\pi} P_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^3 P_{ab}^{(2)}$$

Compare to data



tremendous success

That's all for today