

Perturbative and colorful lectures on **Strong Interactions**

lecture 4/4

Laurent Favart

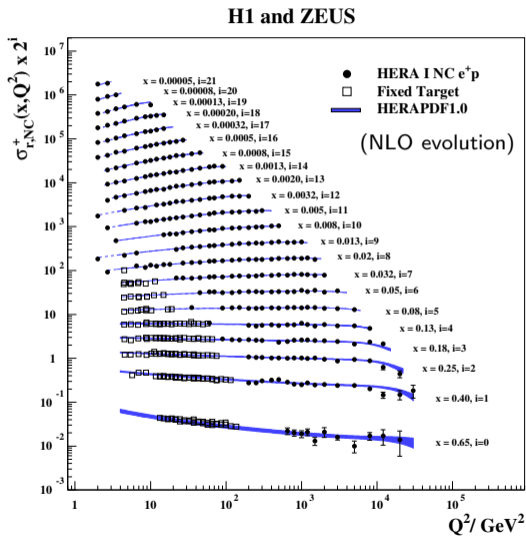
IIHE - Université libre de Bruxelles

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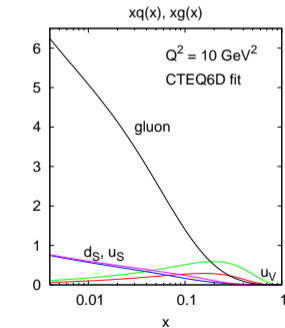
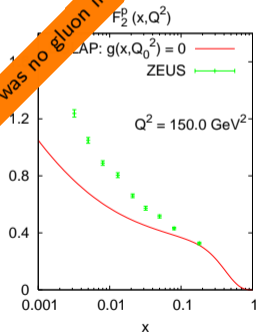
DIS: recap



PDF evolutions: recap

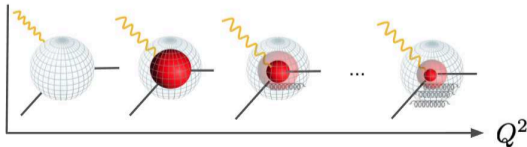
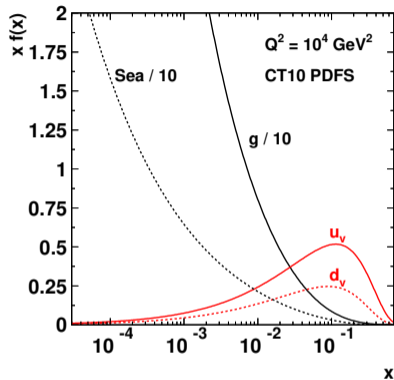
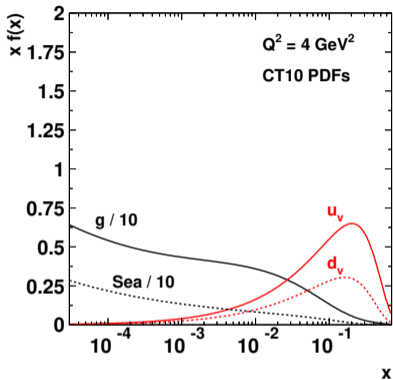
$$q(x, Q^2) = q(x, \mu_F^2) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} q(\xi, \mu_F^2) P_{qq} \left(\frac{x}{\xi} \right) \ln \left(\frac{Q^2}{\mu_F^2} \right) \\ + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} g(\xi, \mu_F^2) P_{qg} \left(\frac{x}{\xi} \right) \ln \left(\frac{Q^2}{\mu_F^2} \right)$$

if there was no gluon in the p



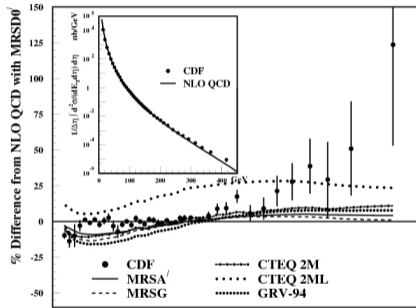
Gluon distribution is **HUGE!**

PDF evolutions: recap



PDF applied to other processes

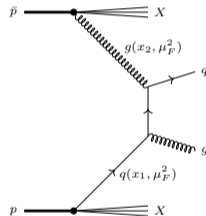
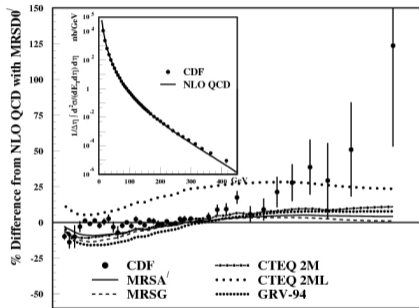
extracted quark and gluon densities can then be used to predict other cross section like $p + \bar{p} \rightarrow 2 \text{ jets}$:



⇒ evidence for quark compositeness in 1995 at TeVatron

PDF applied to other processes

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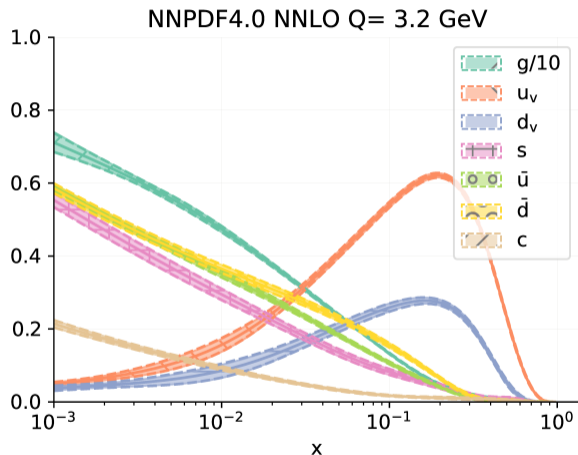
~~\Rightarrow evidence for quark compositeness in 1995 at TeVatron~~

No ! Wrong conclusion because **no PDF uncertainty** was delivered !

here important effect of gluon at large x

PDF uncertainties - state of the art

NNPDF4.0 [2022]

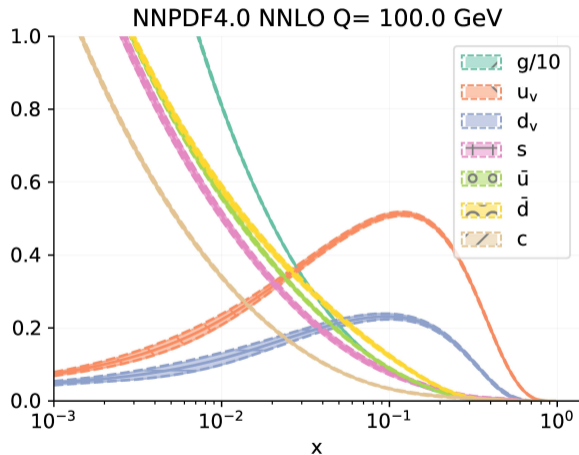


Major activity is translation of experimental errors (and theory uncertainties) into *uncertainty bands on extracted PDFs*.

PDFs with uncertainties allow one to estimate *degree of reliability* of future predictions

PDF uncertainties - state of the art

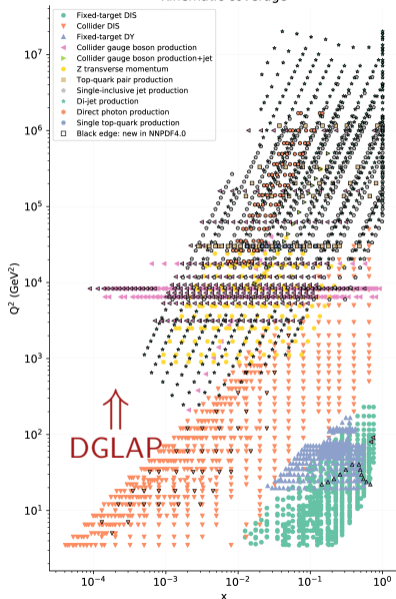
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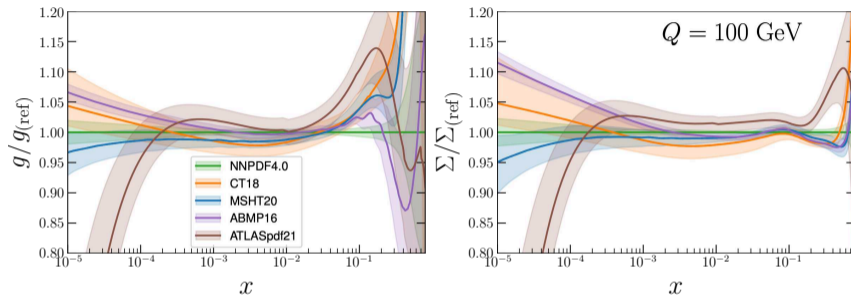
Kinematic coverage



- combined fit of many different measurements (cross sections, ratios, ...) from different colliders & fix target

- most of the measurements for $x \geq 3 \cdot 10^{-4}$

Uncertainties



[arXiv:2203.13923]

- different parametrisations compatible in the well measured phase space
- large differences in the extrapolation region ($x \leq 10^{-4}$)
- large uncertainty on g at $x > 0.1$ and for q at $x \rightarrow 1$

DGLAP

$$\begin{aligned}
 q(x, Q^2) = q(x, \mu_F^2) &+ \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} q(\xi, \mu_F^2) P_{qq} \left(\frac{x}{\xi} \right) \ln \left(\frac{Q^2}{\mu_F^2} \right) \\
 &+ \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} g(\xi, \mu_F^2) P_{qg} \left(\frac{x}{\xi} \right) \ln \left(\frac{Q^2}{\mu_F^2} \right)
 \end{aligned}$$

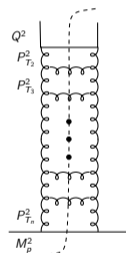
This can be interpreted as the first rung in a set of **ladder diagrams**, whose rungs are **strongly ordered** in $\ln Q^2$.

Including higher orders corresponds to include more rungs in the ladder.

$$Q^2 \simeq p_{T1}^2 \gg p_{T2}^2 \gg p_{T3}^2 \gg \dots$$

⇒ DGLAP equations take into account all contributions proportional to:

$$[\alpha_s(Q^2) \ln \left(\frac{Q^2}{Q_0^2} \right)]^n$$



At small x

$$P_{qq}(z) = \frac{C_F}{2\pi} \left[\frac{1+z^2}{(1-z)} + \frac{3}{2}(1-z) \right] \xrightarrow{z \rightarrow 0} \text{cst}$$

$$P_{gg}(z) = \frac{C_A}{\pi} \left[\frac{1}{z} + \frac{1}{(1-z)} - 2 + z(1-z) \right] \xrightarrow{z \rightarrow 0} 6/z$$

The DGLAP equation get simplified:

$$\frac{\partial xg(x, \mu^2)}{\partial \ln \mu^2} = \frac{3}{\pi b} xg(x, \mu^2)$$

(b includes the α_S dependence). Leads to a solution:

$$xg(x, \mu^2) \sim xg(x, \mu_0^2) \exp \left[2 \sqrt{\frac{6}{b} \ln \ln \left(\frac{\mu^2}{\mu_0^2} \right) \ln \left(\frac{1}{x} \right)} \right]$$

i.e. **double log approximation (DLA)** to DGLAP.

⇒ Fast rise of the cross section at small x .

At still smaller x

$$\ln\left(\frac{1}{x}\right) \gg \ln\left(\frac{Q^2}{Q_0^2}\right)$$

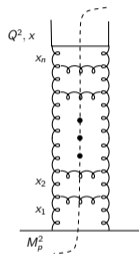
\Rightarrow DGLAP not valid.

Need to consider gluon ladders with repeated iterations of $P_{gg}(z \ll 1)$ dominate, i.e. we have **strong ordering in z** .

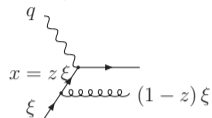
$$x_1 \gg x_2 \gg x_3 \gg \dots \gg x$$

\Rightarrow BFKL (Balitsky-Fadin-Kuraev-Lipatov) equations take into account all contributions proportional to:

$$\left[\alpha_s(Q^2) \ln\left(\frac{1}{x}\right)\right]^n$$



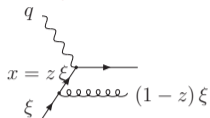
DGLAP/BFKL



$$\frac{d\sigma}{dp_T^2}(z) = \frac{4\pi^2\alpha e_q^2}{s} \frac{1}{p_T^2} \frac{\alpha_S}{2\pi} P_{qq}(z)$$

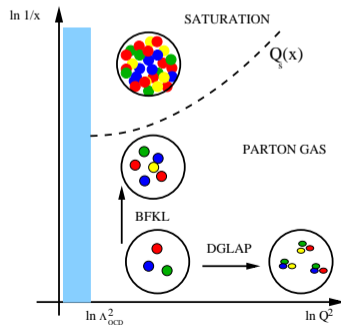
- $\int dP_T^2 \rightarrow \ln Q^2$: DGLAP

DGLAP/BFKL



$$\frac{d\sigma}{dp_T^2}(z) = \frac{4\pi^2\alpha e_q^2}{s} \frac{1}{p_T^2} \frac{\alpha_S}{2\pi} P_{qq}(z)$$

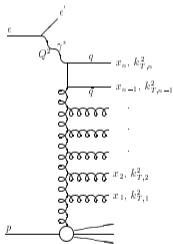
- $\int dP_T^2 \rightarrow \ln Q^2$: DGLAP
- $\int dz \rightarrow \ln 1/x$: BFKL



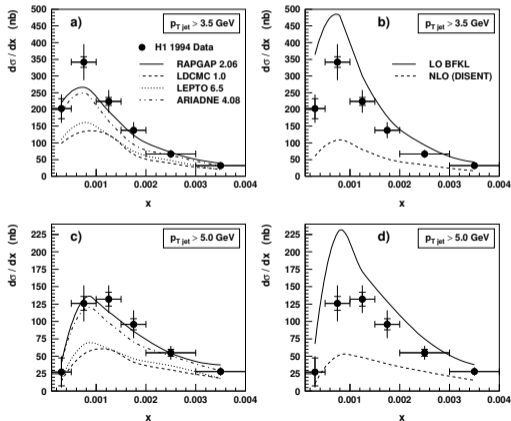
Forward jet production and BFKL

To try to find an evidence of the need of the BFKL dynamic: look at forward jet production.

Select events with $Q^2/P_{Tjet}^2 \sim 1$ to suppress DGLAP region of phase space.



- DGLAP (NLO) prediction too low
- BFKL closer to data



Saturation

Steep rise at small x is problematic: the density gets so large that re-interaction ($gg \rightarrow g$) should take place \rightarrow **saturation**

Saturation has to be expected when Q^2 is such that the recombination cross section times the number of gluons gets close to the hadron transverse size, i.e.

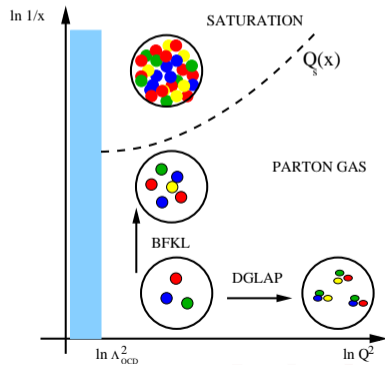
$$\sigma_{gg} N_g \simeq R_{\perp}$$

with $\sigma_{gg} \sim \alpha_S / Q^2$,

$$N_g \sim xg(x, Q^2)$$

saturation should start when:

$$Q_s^2(x) \sim \alpha_S \frac{xg(x, Q^2)}{R_{\perp}} \sim \left(\frac{1}{x}\right)^{\lambda}$$

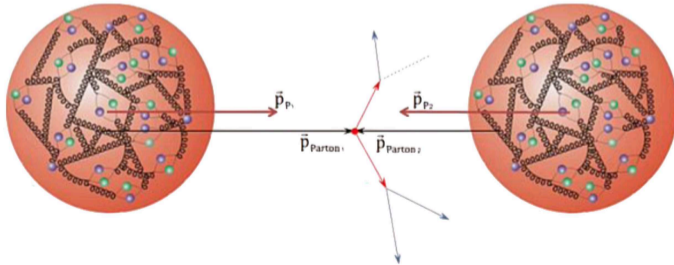


Conclusion of this section and open questions

- In a few decades our understanding of the proton structure has drastically changed
- from a static model with 3 valence quarks to a dynamic object with a high gluon density (\rightarrow what is a colour field ?)
- DGLAP: phenomenal succes of pQCD on a large phase space (4x4 orders of magnetude)
- Do we have to go beyond DGLAP (BFKL dynamics, saturation,...) ? attractive explanations but not fully proven yet.
- many other open questions today:
 - 1) transverse spacial quark and gluon distributions ?
 - 2) correlations between partons ?
 - 3) how is the proton spin mad by the partons ? (orbital angular momentum of q and g)
 - 4) ...

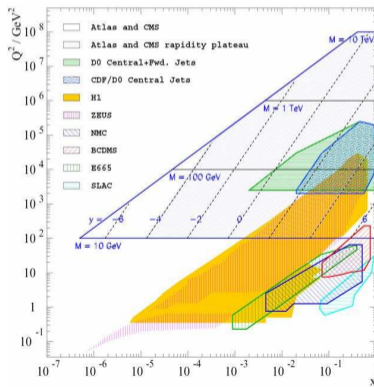
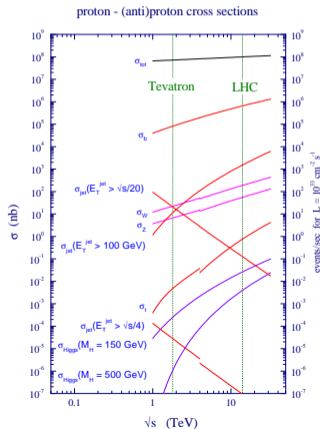
- Part 4 -

hadron - hadron interactions



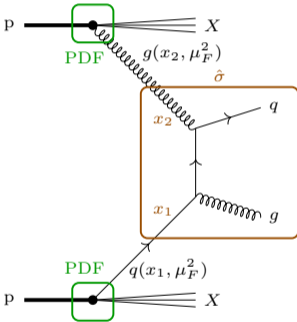
High energy $p - p$ (or $p - \bar{p}$) interactions

- most of the interactions are non perturbative - even at very high \sqrt{s}
- but the available energy makes high momentum transfer possible \rightarrow perturbative interactions (scale = Q^2)

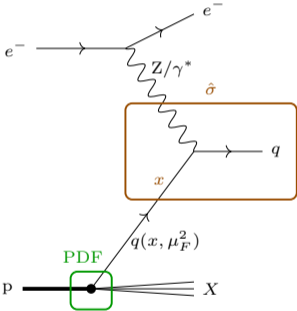


Cross section estimate: the master formula

hadron interactions

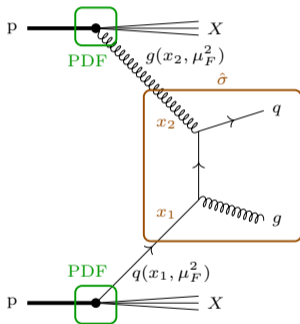


DIS



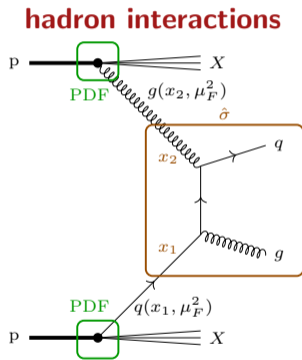
Cross section estimate: the master formula

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$$\sigma_{pp \rightarrow X}(Q) = \sum_{a,b=q,\bar{q},g} \int_0^1 dx_1 dx_2 f_{a/h}(x_1, \mu_F) f_{b/h'}(x_2, \mu_F) \times \hat{\sigma}_{ab \rightarrow X}(Q)(x_1, x_2, Q, \mu_F, \alpha_s(\mu_R)) \times \theta(x_1 x_2 s - Q^2)$$

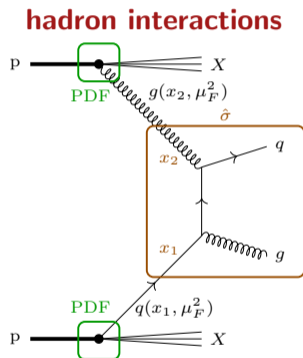
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$$Q^2 = M_{qg}^2 = (p_q + p_g)^2 \\ = 2 E_q E_g (1 - \cos \theta_{qg}) \\ = 4 E_q E_g = 4 x_1 p_1 x_2 p_2 = x_1 x_2 s$$

Cross section estimate: the master formula



$$\sigma_{pp \rightarrow X}(Q) = \sum_{a,b=q,\bar{q},g} \int_0^1 dx_1 dx_2 f_{a/h}(x_1, \mu_F) f_{b/h'}(x_2, \mu_F) \times \hat{\sigma}_{ab \rightarrow X}(Q)(x_1, x_2, Q, \mu_F, \alpha_s(\mu_R)) \times \theta(x_1 x_2 s - Q^2) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{Q^2}\right)$$

$\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{Q^2}\right)$ vanishing term for $Q^2 \rightarrow \infty$

- that can break the factorisation
- called *higher twist*

Multiple Parton Interactions

high energy \rightarrow high parton densities (at low x)

\rightarrow probability of multiple partons scattering increases

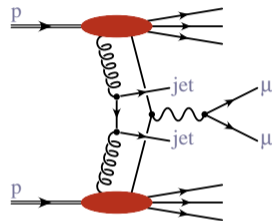
Direct consequence of the composite nature of hadrons.

\rightarrow Out of the frame of the QCD factorisation.

\rightarrow No clear separation with single parton + splitting

\rightarrow non-trivial changes of colour topology

\rightarrow in case of two hard interactions: Double Parton Interaction (DPS)



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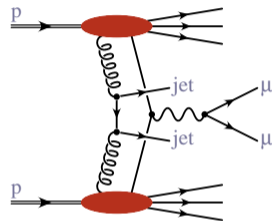
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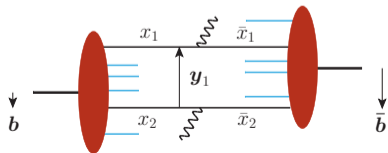
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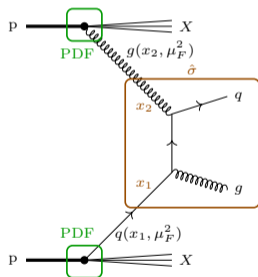
The correlation between the two partons depends on their relative distance:



\Rightarrow possibility to learn about parton location in the proton (like GPDs ?).

Jet production

- The dijet (or total multijet) is the perturbative process with the highest cross section
- first test at a collider with higher energy to test QCD and the presence of new physics



$$\frac{d^3\sigma}{dy_c dy_d dp_T^2} = \frac{1}{16\pi s^2 x_1 x_2} \sum_{a,b,c,d=q,\bar{q},g} f_{a/h}(x_1, \mu_F) f_{b/h'}(x_2, \mu_F) \times \overline{|\mathcal{M}(ab \rightarrow cd)|^2} \frac{1}{1 + \delta_{cd}}$$

δ_{cd} : statistical factor for identical final state

- x_1 and x_2 can be accessed via:

$$\tau = \frac{\hat{s}}{s} = \frac{M_{cd}^2}{s} = x_1 x_2$$

$$y_{cd} = \frac{y_c + y_d}{2} = \frac{1}{2} \ln \frac{x_1}{x_2}$$

Dijet cross section calculation (LO)

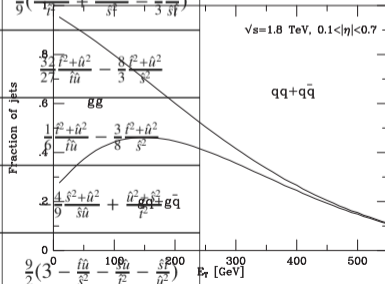
processus	diagrammes	$ \overline{\mathcal{M}} ^2/g^4$
$q q' \rightarrow q q'$		$\frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$
$q q \rightarrow q q$		$\frac{4}{9} \left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} + \frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^2} \right) - \frac{8}{27} \frac{\hat{s}^2}{\hat{u}\hat{t}}$
$q \bar{q} \rightarrow q' \bar{q}'$		$\frac{4}{9} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$
$q \bar{q} \rightarrow q \bar{q}$		$\frac{4}{9} \left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} + \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2} - \frac{2}{3} \frac{\hat{u}^2}{\hat{s}\hat{t}} \right)$
$q \bar{q} \rightarrow g g$		$\frac{8}{27} \frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} - \frac{8}{3} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$
$g g \rightarrow q \bar{q}$		$\frac{1}{6} \frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} - \frac{3}{8} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$
$g q \rightarrow g q$		$\frac{2}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{s}\hat{u}} + \frac{\hat{u}^2}{\hat{t}^2} \frac{q\bar{q} \rightarrow g\bar{q}}$
$g g \rightarrow g g$		$\frac{9}{2} \left(3^0 - \frac{\hat{t}\hat{u}}{\hat{s}^2} - \frac{10\hat{t}\hat{u}}{\hat{t}^2} - \frac{\hat{s}^2}{\hat{u}^2} \right)$

with

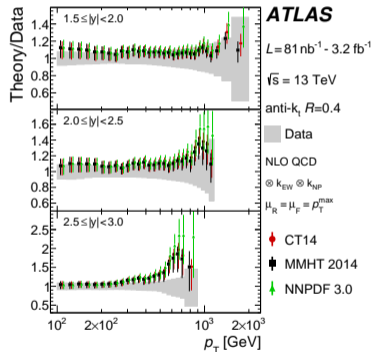
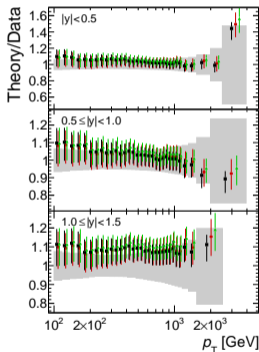
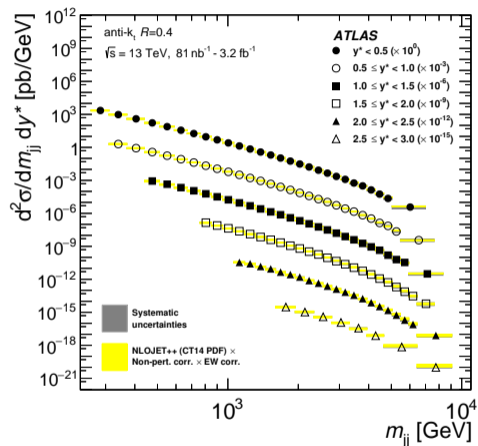
$$\hat{s} = (p_a + p_b)^2$$

$$\hat{t} = (p_a - p_c)^2$$

$$\hat{u} = (p_b - p_c)^2$$



Dijet cross section measurement



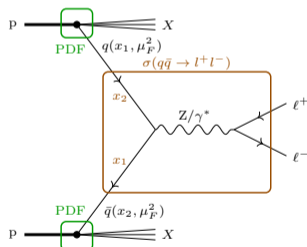
- this measurement constrains further gluon densities at large x

Drell-Yan process

- annihilation of a q with a \bar{q} in hadronic interactions giving a charged lepton pair
- the only process in hadron interaction for which we are **approaching percent level precision** both experimentally and theoretically

at LO, one can derive easily $\sigma_{q\bar{q} \rightarrow l^- l^+}$ from:

$$\frac{d\sigma_{e^- e^+ \rightarrow q\bar{q}}}{d\Omega} = \frac{\alpha^2}{4s} e_q^2 (1 + \cos^2 \theta) = \frac{\alpha^2}{4s} e_q^2 \frac{t^2 + u^2}{s^2}$$



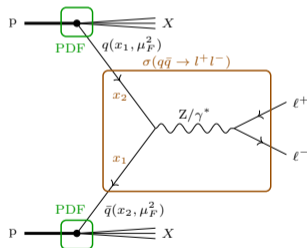
where a sum on final state colors was done, here we should take the average:

$$\frac{d\sigma_{q\bar{q} \rightarrow l^- l^+}}{d\Omega} = \frac{1}{3} \frac{\alpha^2}{4s_{q\bar{q}}} e_q^2 \frac{t^2 + u^2}{s_{q\bar{q}}^2}$$

$$\sigma(q\bar{q} \rightarrow l^- l^+) = \frac{4\pi\alpha^2}{9s_{q\bar{q}}} e_q^2$$

Drell-Yan process

- annihilation of a q with a \bar{q} in hadronic interactions giving a charged lepton pair
- the only process in hadron interaction for which we are **approaching percent level precision** both experimentally and theoretically



$$\frac{d\sigma_{pp \rightarrow l^- l^+ X}}{dQ^2} = \sum_{q, \bar{q}} \int_0^1 dx_1 \int_0^1 dx_2 [q(x_1) \bar{q}(x_2) + (q \leftrightarrow \bar{q})] \times \sigma(q\bar{q} \rightarrow e^- e^+) \delta(Q^2 - s_{q\bar{q}})$$

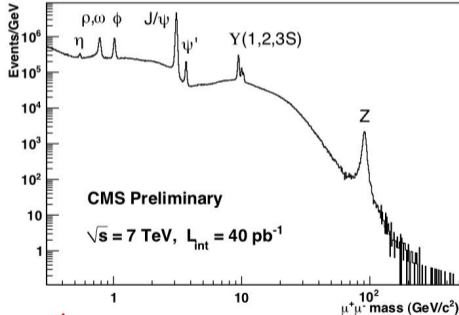
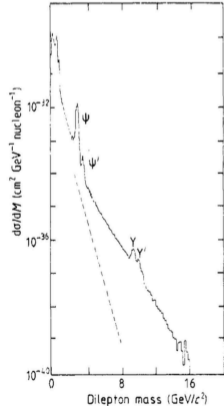
using $\tau = \frac{m_{ll}^2}{s} = \frac{Q^2}{s} = \frac{s_{q\bar{q}}}{s} = x_1 x_2$ and using the δ to get rid of the integral over x_2 :


$$\frac{d\sigma_{pp \rightarrow l^- l^+ X}}{d\tau} = \frac{4\pi\alpha^2}{9s} \frac{1}{\tau} \sum_{q, \bar{q}} e_q^2 \int_{\tau}^1 \frac{dx_1}{x_1} [q(x_1) \bar{q}(\tau/x_1) + (q \leftrightarrow \bar{q})]$$

\Rightarrow scaling (as in DIS)

DY: Mass distribution

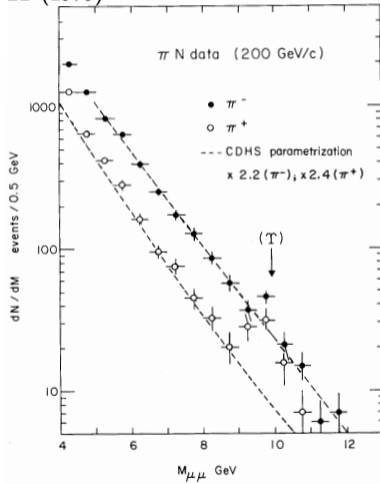
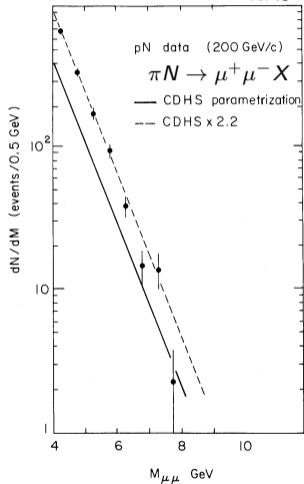
- here we derived only the one γ exchange
- but **resonances** have to be taken into account
- in particular the Z boson at the LHC/TeVatron



 distribution not corrected for acceptance and efficiency effects !

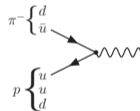
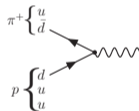
First DY Cross section measurements

NA3 - PLB (1979)



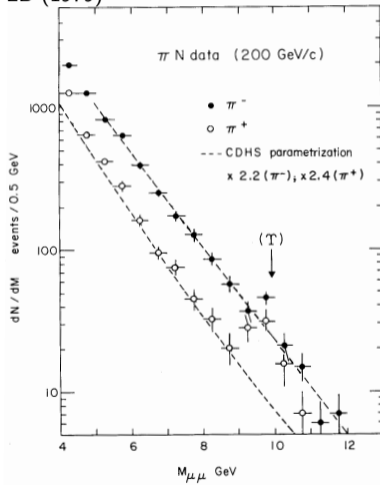
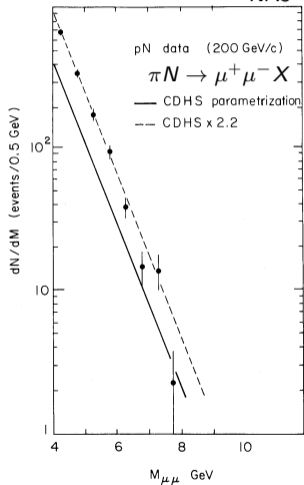
- (out of the resonance regions)
scaling observed ! confirms
 the model i.e. also the QCD
 factorisation

$$- \frac{\sigma(\pi^- N \rightarrow \mu^+ \mu^-)}{\sigma(\pi^+ N \rightarrow \mu^+ \mu^-)} \simeq 2 \text{ as expected}$$



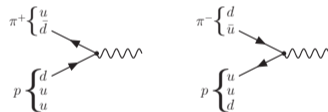
First DY Cross section measurements

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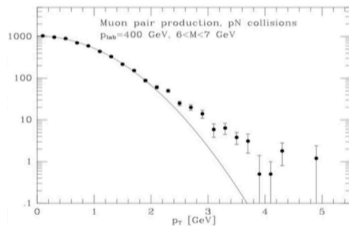


- but the normalisation is wrong by a factor 2.2 !

great success, but...

DY: transverse momentum

- in top of the normalisation problem, the transverse momentum of the lepton pair is not described
- at LO, everything is longitudinal (x of PDF are p longitudinal momentum fraction)



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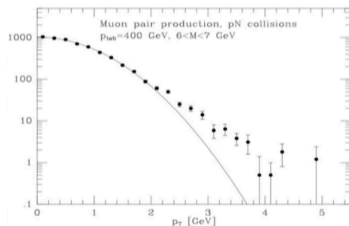
- at LO, everything is longitudinal (x of PDF are p longitudinal momentum fraction)

- low P_T part explained by the **Fermi motion inside the proton**:


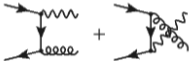
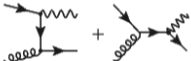
$\Delta p \geq \hbar/2\Delta x \simeq 113 \text{ MeV}$ for each transverse direction and each proton (of $\Delta x = 0.87 \text{ fm}$)

\Rightarrow typically **500 MeV** (fit to date gives 760 MeV)


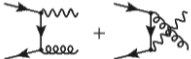
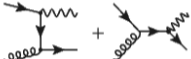
- missing NLO !



DY at NLO

LO	NLO	
	processus d'annihilation	processus QCD
$q + \bar{q} \rightarrow \gamma^*$	$q + \bar{q} \rightarrow g + \gamma^*$	$q + g \rightarrow q + \gamma^*$
		
1	$16\pi^2\alpha_S\alpha\frac{8}{9}\left[\frac{\hat{u}^2}{\hat{t}^2} + \frac{\hat{t}^2}{\hat{u}^2} + \frac{2M^2\hat{s}}{\hat{u}\hat{t}}\right]$	$16\pi^2\alpha_S\alpha\frac{1}{3}\left[-\frac{\hat{t}^2}{\hat{s}^2} - \frac{\hat{s}^2}{\hat{t}^2} - \frac{2M^2\hat{u}}{\hat{s}\hat{t}}\right]$

DY at NLO

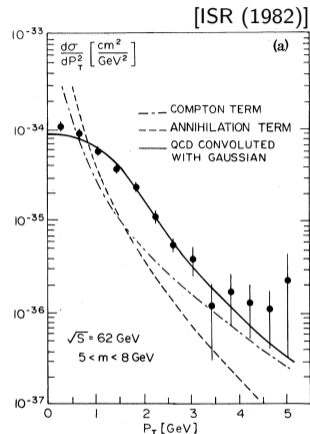
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⇒ nicely describes the large P_T distribution

⇒ nicely describes the normalisation

- large NLO effect because a new type of diagram comes in, furthermore with (huge) gluon densities

complete success



DY constrains on PDF

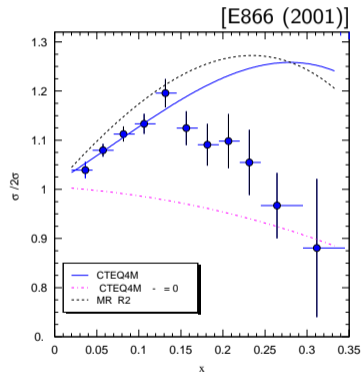
- assuming hypercharge symmetry such as $u = u_p = d_n$ and $\bar{d} = \bar{d}_p = \bar{u}_n$,
- assuming there are only 2 flavours :

$$\sigma^{pp} \sim \frac{4}{9} u(x_1) \bar{u}(x_2) + \frac{1}{9} d(x_1) \bar{d}(x_2)$$

$$\sigma^{pn} \sim \frac{4}{9} u(x_1) \bar{d}(x_2) + \frac{1}{9} d(x_1) \bar{u}(x_2)$$

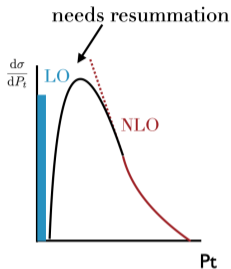
the ratio, using p and deuterium target:

$$\frac{\sigma^{pd}}{2\sigma^{pp}} = \frac{\left(1 + \frac{1}{4} \frac{d(x_1)}{u(x_1)}\right)}{\left(1 + \frac{1}{4} \frac{d(x_1)}{u(x_1)} \frac{\bar{d}(x_2)}{\bar{u}(x_2)}\right)} \left(1 + \frac{\bar{d}(x_2)}{\bar{u}(x_2)}\right) \simeq 1 + \frac{\bar{d}(x_2)}{\bar{u}(x_2)}$$

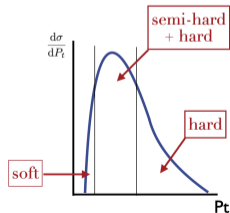
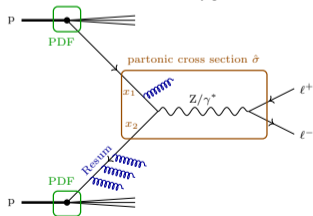


\Rightarrow the sea distributions of \bar{u} and \bar{d} are different ! Surprise !

DY: state of the art

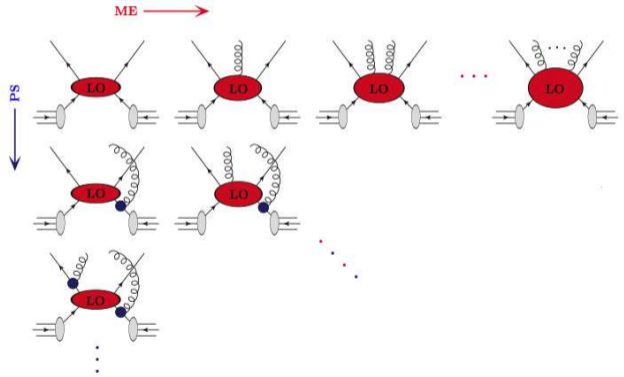


- the NLO contribution **diverges** at low P_t
- a realistic description of the P_t spectrum requires a **resummation** of many **gluon** radiation
- can be done in different ways:
 - **analytic calculation**
 - Monte Carlo Parton Showers (PS)
 - PDF \rightarrow Transverse Momentum Distributions (TMDs)



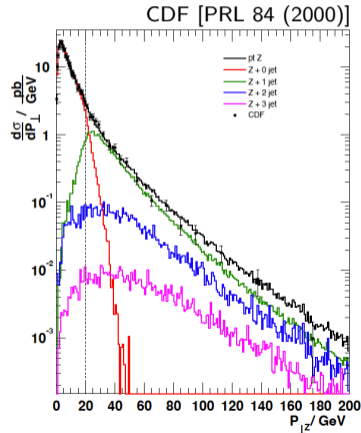
Multileg Monte Carlo

- to compute NN...NNLO is presently not possible
- several modern Monte Carlos make generate **separated samples of LO ME** for a given number of parton in the final state
- **PS** is added on each colored branch of each event
- globally: ME \rightarrow large P_T , PS \rightarrow small P_T
- **merging** procedure is done avoiding double counting
- samples are put together
- **drawback**: PS have fitted parameters they depend on $\sqrt{s} \rightarrow$ problem somewhere !



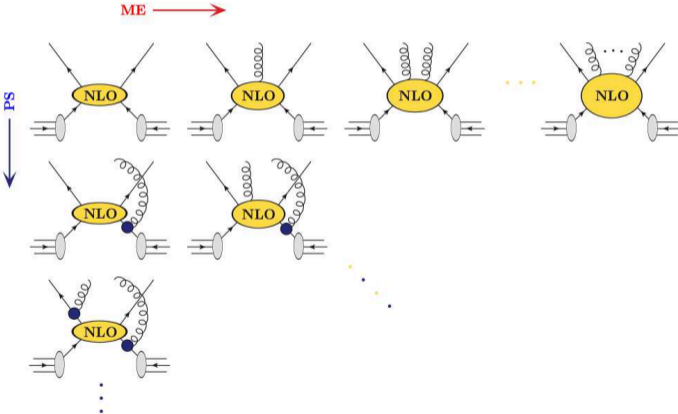
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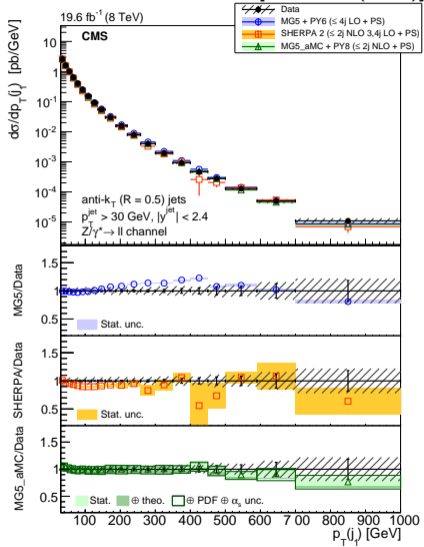
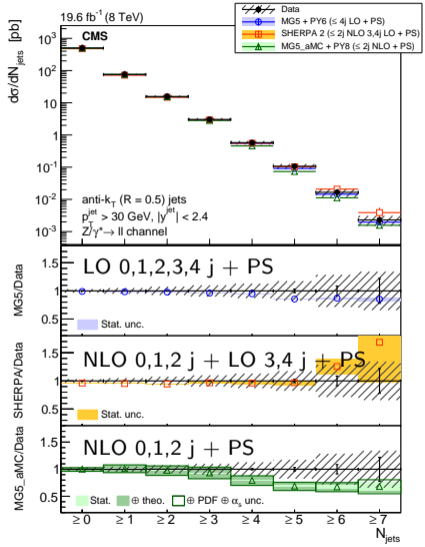
NLO multileg Monte Carlo

- some MC push the complexity to including NLO ME in a multileg approach:



$Z + jet(s)$

- need ME with many partons to described high jet multiplicity
- need ME at NLO to describe well the P_t shapes (jet or Z), at large P_t
- approaching high precision, in a large phase space and for up to 2 jet multiplicities



TMDs: Transverse Momentum Distributions

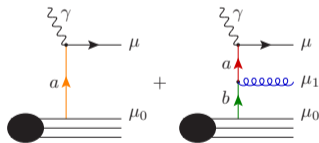
- PDF: $f_a(x, \mu^2)$ are purely longitudinal, $a = q, g$
- TMDs: $f_a(x, \vec{k}_t, \mu^2)$ include a transverse component
- \exists several different approaches. Example here PB TMDs: *Parton Branching TMDs*

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- \exists several different approaches. Example here PB TMDs: *Parton Branching TMDs*
- idea of PB TMDs: construct iteratively \vec{k}_t purely dynamically

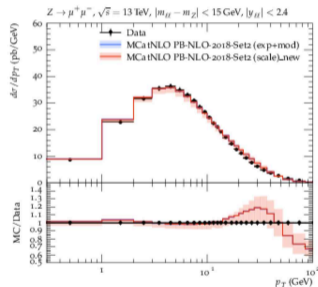
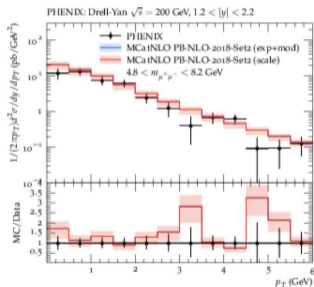
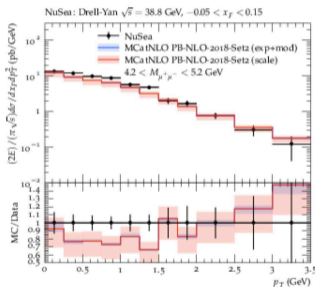
$$f_a(x, \mu^2) = f_a(x, \mu_0^2) \Delta_a(\mu^2, \mu_0^2) + \int_{\mu_0^2}^{\mu^2} \frac{d\mu_1^2}{\mu_1^2} \Delta_a(\mu^2, \mu_1^2) \sum_b \int_x^{z_M} \frac{dz}{z} \frac{\alpha_S}{2\pi} P_{ab}^R(z) f_b(x/z, \mu_1^2) \Delta_b(\mu_1^2, \mu_0^2)$$

$\Delta_a(\mu_2^2, \mu_1^2)$ is Sudakov form factor, i.e. the probability to have a scale evolution $\mu_1^2 \rightarrow \mu_2^2$ without parton radiation



PBTMDs

- an **iterative procedure** is applied, keeping in memory the kinematic at each step
- and in particular the **transverse momenta**, choosing: $p_T^2 = (1 - z)^2 \mu_i^2$
- PBTMDs are obtained from **fit to HERA data**, then predicts Drell-Yan cross sections:



[Eur.Phys.J.C 80 (2020) 7, 598]

- only 1 parameter of non-pert. origin: intrinsic p_t

great success !

Conclusion and Open questions

- The TeVatron & LHC opened a new range in energy
- allowed study processes at high scales and multijet production
- huge progresses have been achieved on all aspects
- a precise prediction (at % level) remains a challenge for many observables
- they are needed to measure the Higgs production (very close to Drell-Yan) and decay as precisely as possible
- and to put constrains on new physics

General conclusions

- the understanding of the strong interactions has started about 50 years ago
- the strong interaction is responsible for a very large diversity of phenomena and reactions (confinement of hadrons and of nuclei, nuclear physics, asymptotic freedom, q-g plasma, particle interactions,...)
- we now reach the 1 percent precision level for some processes
- there are still many things to be understood and probably many surprises to come...